1. a) For example the assignment $\mathsf{v}(\mathsf{p}) = \mathsf{F}$ and $\mathsf{v}(\mathsf{q}) = \mathsf{T}$ suffices to show that A is satisfiable; the assignment $\mathsf{v}(\mathsf{p}) = \mathsf{T}$ and $\mathsf{v}(\mathsf{r}) = \mathsf{T}$, yields that B is not a tautology.

   b) We translate B into clause form: $\mathcal{C} = \{\neg \mathsf{p} \lor \mathsf{q}, \mathsf{p} \lor \mathsf{q}, \neg \mathsf{q} \lor \neg \mathsf{q}\}$. Then it suffices to apply two factoring steps and two resolution steps to derive the empty clause. Hence the formula is refutable.

   c) We show that the formula $\neg C$ is inconsistent. Transformation of $\neg C$ yields the following clause set:

$$\mathsf{p} \quad \mathsf{r} \quad \neg\mathsf{p} \lor \mathsf{q} \quad \neg\mathsf{r} \lor \mathsf{s} \quad \neg\mathsf{q} \lor \neg\mathsf{s}$$

   The following resolution proof shows that formula $\neg C$ is unsatisfiable:

$$\dfrac{\dfrac{\neg\mathsf{p} \lor \mathsf{q} \quad \mathsf{p}}{\mathsf{q}} \qquad \dfrac{\dfrac{\neg\mathsf{r} \lor \mathsf{s} \quad \mathsf{r}}{\mathsf{s}} \quad \neg\mathsf{q} \lor \neg\mathsf{s}}{\neg\mathsf{q}}}{\Box}$$

2. a) See Definition 3.9 on page 19.

   b) Consider a language $\mathcal{L} = \{\mathsf{P}\}$ that consists only of a unary predicate constant. For $\mathcal{L}$, we define two structues $\mathsf{A} = (\mathbb{N}, a_{\mathsf{A}})$ and $\mathcal{B} = (\mathbb{N}, a_{\mathcal{B}})$ over the natural numbers, where the mapping a_{A} intepretes P as the set of all numbers, but $a_{\mathcal{B}}(\mathsf{P}) = \varnothing$. By the above definition we trivially have $\mathsf{A} \cong_1 \mathcal{B}$, but clearly $\mathsf{A} \models \forall x\ \mathsf{P}(x)$, while $\mathcal{B} \not\models \forall x\ \mathsf{P}(x)$.

3. a)
   ① $\forall x\ (\mathsf{Dragon}(x) \land \forall y\ (\mathsf{Child}(y, x) \to \mathsf{Happy}(y)) \to \mathsf{Happy}(x))$.
   ② $\forall x\ (\mathsf{Dragon}(x) \to (\mathsf{Fly}(x) \leftrightarrow (\exists y\ \mathsf{Ancestor}(y, x) \land \mathsf{Fly}(y))))$.
   ③ $\forall x\ (\mathsf{Dragon}(x) \land (\exists y\ \mathsf{Child}(x, y) \land (\mathsf{colour}(y) = \mathsf{red})) \to (\mathsf{colour}(x) = \mathsf{green}))$.
   ④ $\forall x\ (\mathsf{Dragon}(x) \land (\mathsf{colour}(x) = \mathsf{green})) \to \neg\mathsf{Spitfire}(x)$.
   ⑤ $\exists x\ (\mathsf{Dragon}(x) \land (\mathsf{colour}(x) = \mathsf{red}) \land \neg\mathsf{Fly}(x))$.

   b) We define a suitable structure $\mathsf{A} = (A, a)$ where $A = \{green, red, grisu\}$ and the mapping a is defined as follows:

   - $a(\mathsf{green}) = green$, $a(\mathsf{red}) = red$
   - $a(\mathsf{colour}) = f \colon \{green, red, grisu\} \mapsto red$, that is all function symbols are mapped to the constant function that always returns red.
   - $a(\mathsf{Dragon}) = a(\mathsf{Happy}) = \{grisu\}$ and $a(\mathsf{Fly}) = a(\mathsf{Child}) = a(\mathsf{Ancestor}) = a(\mathsf{Spitfire}) = \varnothing$.

Let l be an arbitrary assignment and $\mathcal{M} = (\mathsf{A}, l)$. Then $\mathcal{M} \models F$ holds for each of the five sentences above and thus the formalisation is satisfiable.

4. a) The formula is unsatisfiable.

   b) In the *ordered* resolution proofs given below the most general unifier employed is written to the right of each applied inference. First we derive the clause $\mathsf{R}(\mathsf{a}, y)$ as follows.

$$\dfrac{\dfrac{\mathsf{P}(x) \lor \mathsf{Q}(x) \lor \mathsf{R}(x,y) \quad \neg\mathsf{P}(x)}{\mathsf{Q}(x) \lor \mathsf{R}(x,y)} \qquad \neg\mathsf{Q}(\mathsf{a})}{\mathsf{R}(\mathsf{a},y)}\ \sigma = \{x \mapsto \mathsf{a}\}$$

Using this deduction Π, we derive the empty clause.

$$\dfrac{\begin{matrix}\Pi \\ \mathsf{R}(\mathsf{a},y)\end{matrix} \qquad \dfrac{\dfrac{\dfrac{\mathsf{S}(\mathsf{a},y) \lor \neg\mathsf{R}(\mathsf{a},y) \lor \mathsf{S}(x,\mathsf{b})}{\mathsf{S}(\mathsf{a},\mathsf{b}) \lor \neg\mathsf{R}(\mathsf{a},\mathsf{b})}\ \sigma_1 \qquad \neg\mathsf{S}(\mathsf{a},\mathsf{b}) \lor \neg\mathsf{R}(\mathsf{a},\mathsf{b})}{\neg\mathsf{R}(\mathsf{a},\mathsf{b}) \lor \neg\mathsf{R}(\mathsf{a},\mathsf{b})}}{\neg\mathsf{R}(\mathsf{a},\mathsf{b})}}{\square}\ \sigma_2$$

Here $\sigma_1 := \{x \mapsto \mathsf{a}, y \mapsto \mathsf{b}\}$ and $\sigma_2 := \{y \mapsto \mathsf{b}\}$.

5.

statement	**yes**	**no**
Let $\mathcal{I}_1, \mathcal{I}_2$ be interpretations such that the respective universes coincide and suppose $\mathcal{I}_1, \mathcal{I}_2$ coincide on the constants in the closed formula F. Then $\mathcal{I}_1 \models F$ iff $\mathcal{I}_2 \models F$.	✓	
Let $\mathcal{A}$, $\mathcal{B}$ be structures and $\mathcal{A} \cong \mathcal{B}$. Then for every sentence F we have $\mathcal{A} \models F$ iff $\mathcal{B} \models F$.	✓	
For all formulas F and all sets of formulas $\mathcal{G}$ we have that $\mathcal{G} \models F$ iff $\mathsf{Sat}(\mathcal{G} \cup \{\neg F\})$.		✓
Suppose $\mathcal{G}$ is a set of formulas and $\mathcal{G} \models F$. Then there exists a finite subset $\mathcal{G}_0 \subseteq \mathcal{G}$ such that $\mathcal{G}_0 \models F$.	✓	
Let A, B be sets such that there exists a bijection m between them. Then if $\mathcal{A}$ is a structure with domain A, there exists a structure $\mathcal{B}$ with domain B such that $\mathcal{A} \cong \mathcal{B}$.	✓	
If formula F has a finite model, then F has a model in the domain $\{0, 1, 2, \ldots, n\}$.	✓	
If a set of formulas $\mathcal{G}$ has an infinite model, then $\mathcal{G}$ has no countable infinite model.		✓
There exists a first-order sentence F and a set of clauses $\mathcal{C} = \{C_1, \ldots, C_m\}$ such that $F \approx \forall x_1 \ldots \forall x_n (C_1 \wedge \cdots \wedge C_m)$ does not hold.	✓	
A second-order interpretation consists of a second-order structure $\mathcal{A}$ and a first-order environment.	✓	
Reachability in directed graphs is expressible as a second-order formula.	✓	