1. Consider the following puzzles (usually called *Who killed Aunt Agatha*):

   Someone who lives in Dreadbury Mansion killed Aunt Agatha. Agatha, the butler, and Charles live in Dreadbury Mansion, and are the only people who live therein. A killer always hates his victim, and is never richer than his victim. Charles hates no one that Aunt Agatha hates. Agatha hates everyone except the butler. The butler hates everyone not richer than Aunt Agatha. The butler hates everyone Aunt Agatha hates. No one hates everyone. Agatha is not the butler.

   a) Formalise the assumptions of the puzzle in first-order logic over whatever language you see fit. (10 pts)

   b) Formalise the question *Who killes Aunt Agatha* and solve it using your formalisation. (10 pts)

2. Clarify whether the following formulas are valid sentences in *intuitionistic logic*. Recall, for example, that natural deduction of intuitionistic logic simply omits the rule for double negation.

   a) $(\neg A \vee B) \rightarrow (A \rightarrow B)$. (5 pts)

   b) $\exists x P(x) \rightarrow \exists y(\exists x P(x) \rightarrow P(y))$. (5 pts)

   c) $\exists x(A \rightarrow B(x)) \rightarrow (A \rightarrow \exists x B(x))$. (5 pts)

   d) $\neg\neg\neg A \equiv \neg A$. (5 pts)

3. Proof the following lemma:

   Lemma. Suppose that $\mathcal{L}$ is free of the equality symbol. . Let $\mathcal{L}^+$ denote an extension of $\mathcal{L}$ by infinitely many individual constants. Further, let $\mathcal{G}$ be a set of formulas (of $\mathcal{L}$) admitting the closure properties (with respect to $\mathcal{L}^+$). Then there exists an interpretation $\mathcal{M}$ such that every element of the domain of $\mathcal{M}$ is the denotation of a term (of $\mathcal{L}^+$).

   (20 pts)

4. Consider the following sentence F:

   $$(\forall x \forall y P(x,y) \wedge \forall u \forall v(P(u,v) \rightarrow R(u))) \rightarrow \forall z R(z) \ .$$

   a) Transform F into its Skolem normal form G. (5 pts)

   b) Clarify whether G is satisfiable, unsatisfiable, or valid. What conclusions can be drawn for F from the result? (5 pts)

   c) Transform $\neg F$ into its Skolem normal form H and repeat b) analogously. (10 pts)

5. Determine whether the following statements are true or false. Every correct answer is worth 2 points. (20 pts)

statement	**yes**	**no**
Consider propositional logic. Then $A_1, \dots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \dots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.	☐	☐
Natural deduction for propositional logic is sound and complete.	☐	☐
An interpretation $\mathcal{I}$ is a pair $\mathcal{A} = (A, a)$ such that (i) A is a non-empty set, called *domain* and (ii) the mapping a associates constants with the domain.	☐	☐
For all formulas F and all sets of formulas $\mathcal{G}$ we have that $\mathcal{G} \models F$ iff $\neg\,\mathsf{Sat}(\mathcal{G} \cup \{\neg F\})$.	☐	☐
Let $\mathcal{A}$, $\mathcal{B}$ be structures such that $\mathcal{A} \cong \mathcal{B}$ and let ℓ be an environment. Then for every formula F we have $(\mathcal{A}, \ell) \models F$ iff $(\mathcal{B}, \ell) \models F$.	☐	☐
The set S of all consistent set of formulas has the satisfaction properties.	☐	☐
If a set of formulas $\mathcal{G}$ has arbitrarily large models, then it has a countable infinite model.	☐	☐
For any formula F there exists a formula G such that G does neither contain individual or function constants nor equality and $F \approx G$.	☐	☐
Let $\mathcal{K}$ be a Δ-elementary class of structures. Then there exists a subclass $\mathcal{K}^\infty \subseteq \mathcal{K}$ of structures in $\mathcal{K}$ with infinite domain which is not elementary.	☐	☐
Existential second-order logic is closed under negation.	☐	☐