1. a) *Solution.* For expressivity, predicated have been given meaningful names.

(1) $\exists x\ \mathsf{Lives}(x) \land \mathsf{Killed}(x, \mathsf{agatha})$

(2) $\mathsf{Lives}(\mathsf{agatha})$

(3) $\mathsf{Lives}(\mathsf{butler})$

(4) $\mathsf{Lives}(\mathsf{charles})$

(5) $\forall x(\mathsf{Lives}(x) \to (x = \mathsf{agatha} \lor x = \mathsf{butler} \lor x = \mathsf{charles}))$

(6) $\forall xy(\mathsf{Killed}(x, y) \to \mathsf{Hates}(x, y))$

(7) $\forall xy(\mathsf{Killed}(x, y) \to \neg\mathsf{Richer}(x, y))$

(8) $\forall x(\mathsf{Hates}(\mathsf{agatha}, x) \to \neg\mathsf{Hates}(\mathsf{charles}, x))$

(9) $\forall x(x \neq \mathsf{butler} \to \mathsf{Hates}(\mathsf{agatha}, x))$

(10) $\forall x(\neg\mathsf{Richer}(x, \mathsf{agatha}) \to \mathsf{Hates}(\mathsf{butler}, x))$

(11) $\forall x(\mathsf{Hates}(\mathsf{agatha}, x) \to \mathsf{Hates}(\mathsf{butler}, x))$

(12) $\forall x \exists y \neg\mathsf{Hates}(x, y)$

(13) $\mathsf{agatha} \neq \mathsf{butler}$

□

b) *Solution.* Let Γ contain the axioms in (1)–(13) in a), then the question is formalisable as the following consequence:

$$\Gamma \models \exists \mathsf{Killed}(x, \mathsf{agatha}) \ .$$

Using semantic argumentation we see that $\Gamma \models \mathsf{Killed}(\mathsf{agatha}, \mathsf{agatha})$. □

2. a) *Solution.* We argue in the sequent calculus for intuitionistic logic:

$$\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{A, \neg A \vdash} \quad B \vdash B}{\neg A \lor B, A \vdash B}}{\neg A \lor B \vdash A \to B}}{\vdash \neg A \lor B \to A \to B}$$

□

b) *Solution.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x), \exists x P(x) \vdash P(x)}}{P(x) \vdash \exists x P(x) \to P(x)}}{P(x) \vdash \exists y(\exists x P(x) \to P(y))}}{\exists x P(x) \vdash \exists y(\exists x P(x) \to P(y))}}{\vdash \exists x P(x) \to \exists y(\exists x P(x) \to P(y))}$$

$\square$

c) *Solution.*

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A \quad B(x) \vdash B(x)}{A \to B(x), A \vdash B(x)}}{A \to B(x), A \vdash \exists x B(x)}}{A \to B(x) \vdash (A \to \exists x B(x))}}{\exists x(A \to B(x)) \vdash (A \to \exists x B(x))}}{\vdash \exists x(A \to B(x)) \to (A \to \exists x B(x))}$$

$\square$

d) *Solution.* We only show one direction, the other direction is similar.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash A}{\neg A, A \vdash}}{A \vdash \neg\neg A}}{A, \neg\neg\neg A \vdash}}{\neg\neg\neg A \vdash \neg A}}{\vdash \neg\neg\neg A \to \neg A}$$

$\square$

3. *Solution.* Simplification of the the proof of Lemma 4.4 in the lecture notes, plus extension of the term model to functions. For the latter the following setting suffices:

$$f^{\mathcal{M}}(t_1, \dots, t_n) := f(t_1, \dots, t_n) \; .$$

$\square$

4. a) *Solution.*

$$G := \forall z((\neg P(a,b) \vee P(c,d) \vee R(z)) \wedge (\neg P(a,b) \vee \neg R(c) \vee R(z))) \; .$$

$\square$

b) *Solution.* G is satisfiable, and from this we can only conclude that F is satisfiable. $\square$

c) *Solution.*

$$H := \forall xyuv(P(x,y) \wedge (\neg P(u,v) \vee R(u)) \wedge \neg R(a)) \; .$$

H is unsatisfiable, and thus $\neg F$ is unsatisfiable, and thus F is valid. $\square$

5.

Solution.

statement	**yes**	**no**
Consider propositional logic. Then $A_1, \dots, A_n \models B$, asserts that $\mathsf{v}(B) = \mathsf{T}$, whenever there exists $i \in \{1, \dots, n\}$ such that $\mathsf{v}(A_i) = \mathsf{T}$, for any assignment v.	$\square$	$\boxed{\checkmark}$
Natural deduction for propositional logic is sound and complete.	$\boxed{\checkmark}$	$\square$
An interpretation $\mathcal{I}$ is a pair $\mathcal{A} = (A, a)$ such that (i) A is a non-empty set, called *domain* and (ii) the mapping a associates constants with the domain.	$\square$	$\boxed{\checkmark}$
For all formulas F and all sets of formulas $\mathcal{G}$ we have that $\mathcal{G} \models F$ iff $\neg\,\mathsf{Sat}(\mathcal{G} \cup \{\neg F\})$.	$\boxed{\checkmark}$	$\square$
Let $\mathcal{A}$, $\mathcal{B}$ be structures such that $\mathcal{A} \cong \mathcal{B}$ and let ℓ be an environment. Then for every formula F we have $(\mathcal{A}, \ell) \models F$ iff $(\mathcal{B}, \ell) \models F$.	$\square$	$\boxed{\checkmark}$
The set S of all consistent set of formulas has the satisfaction properties.	$\boxed{\checkmark}$	$\square$
If a set of formulas $\mathcal{G}$ has arbitrarily large models, then it has a countable infinite model.	$\boxed{\checkmark}$	$\square$
For any formula F there exists a formula G such that G does neither contain individual or function constants nor equality and $F \approx G$.	$\square$	$\boxed{\checkmark}$
Let $\mathcal{K}$ be a Δ-elementary class of structures. Then there exists a subclass $\mathcal{K}^\infty \subseteq \mathcal{K}$ of structures in $\mathcal{K}$ with infinite domain which is not elementary.	$\boxed{\checkmark}$	$\square$
Existential second-order logic is closed under negation.	$\square$	$\boxed{\checkmark}$

$\square$