Problem Set 2 (for November 9)

Marked problems will be discussed on November 9; for solutions to the other problems please contact Georg Moser.

- Let F be a propositional formula, where zero, one or more subformulas G are replaced by a logical equivalent formula G'. Then we obtain a formula F' that is logical equivalent to the formula F.
  1. Give a precise definition of this process of substitution.
  2. Show the correctness of the claim. (∗)

- Indicate the form of the following argument—traditionally called 'syllogism in Felapton'—using first-order formulas:
  1. No centaurs are allowed to vote.
  2. All centaurs are intelligent beings.
  3. Therefore, some intelligent beings are not allowed to vote. (∗)

- Let $\mathcal{L} = \{F, P, =\}$, where F is unary, P is binary and let $\mathcal{A}$ be a structure whose domain are sets of persons, such that $P(a, b)$ denotes "a is parent of b" and F "female". Give informal explanations of the following formulas:
  1. $\exists z \exists u \exists v (u \neq v \wedge P(u, b) \wedge P(v, b) \wedge P(u, z) \wedge P(v, z) \wedge P(z, a) \wedge \neg F(b))$
  2. $\exists z \exists u \exists v (u \neq v \wedge P(u, a) \wedge P(v, a) \wedge P(u, z) \wedge P(v, z) \wedge P(z, b) \wedge F(b))$

- Consider the following sentences:

  ① Each man is happy if all its children are happy.
  ② Men have green eyes if at least two of their ancestors have green eyes.
  ③ A man is really small if one of its parents is large.
  ④ Large men are not really small.
  ⑤ There are men with brown eyes that are large. (∗)

  For each of the sentences above, give a first-order formula that formalises it. Use the following constants, functions and predicates:

  – constants: green, red.
  – functions: $\mathsf{eyecolour}(a)$.
  – predicates: $\mathsf{Man}(a)$, $\mathsf{Large}(a)$, $\mathsf{ReallySmall}(a)$, $\mathsf{Happy}(a)$, $\mathsf{Child}(a, b)$ ("a is child of b"), $\mathsf{Ancestor}(a, b)$ ("a is ancestor of b"), $=$.

- Show that the formalisation in the previous problem is satisfiable