First Exam
Automated Reasoning, LVA 703608

February 28, 2014

Name: **Studentnumber**:

The exam consists of 5 exercises with a total of 100 points. Please fill our your name and credentials *before* you start the exam.

1	2	3	4	5	Sum

1. Consider the following formula F

$$\exists x \forall y \forall z \forall w \, (\mathsf{P}(y) \vee \mathsf{Q}(z) \vee \mathsf{R}(w) \rightarrow \mathsf{P}(x) \vee \mathsf{Q}(x) \vee \mathsf{R}(x)) \ .$$

   a) Provide a *free-variable* semantic tableaux proof of F (10 pts)

   b) Provide a resolution proof of F using the unrefined resolution operator. (10 pts)

   c) Compare the different derivations and explain the inefficiency of the obtained semantic tableaux proof. (5 pts)

2. Consider the formula F from Exericse 1.

   a) What is $\mathsf{rsk}(F)$? (8 pts)

   b) Can the result of $\mathsf{rsk}(F)$ be improved somehow? (7 pts)

3. Prove the following variant of the lifting lemma for *ordered* resolution.

   Lemma. Let τ_1 and τ_2 be ground substitutions and consider the following ground ordered resolution step:

$$\frac{C\tau_1 \vee A\tau_1 \quad D\tau_2 \vee \neg B\tau_2}{C\tau_1 \vee D\tau_2} \ ,$$

   where $A\tau_1 = B\tau_2$ and $B\tau_1$ ($A\tau_1$) is (strictly) maximal wrt. $D\tau_2$ ($C\tau_1$). Then there exists a mgu σ of A and B, such that σ is more general then τ_1 and τ_2 and the following *ordered* resolution step is valid:

$$\frac{C \vee A \quad D \vee \neg B}{(C \vee D)\sigma} \ .$$

   (18 pts)

4. Recall the superposition calculus:

$$\frac{C \vee A \quad D \vee \neg B}{(C \vee D)\sigma} \qquad \frac{C \vee A \vee B}{(C \vee A)\sigma}$$

$$\frac{C \vee s = t \quad D \vee \neg A[s']}{(C \vee D \vee \neg A[t])\sigma} \qquad \frac{C \vee s = t \quad D \vee A[s']}{(C \vee D \vee A[t])\sigma}$$

$$\frac{C \vee s = t \quad D \vee u[s'] \neq v}{(C \vee D \vee u[t] \neq v)\sigma} \qquad \frac{C \vee s = t \quad D \vee u[s'] = v}{(C \vee D \vee u[t] = v)\sigma}$$

$$\frac{C \vee s \neq t}{C\sigma} \qquad \frac{C \vee u = v \vee s = t}{(C \vee v \neq t \vee u = t)\sigma} \ ,$$

   (In addition certain conditions apply). Consider the following clause set $\mathcal{C}$:

$$\mathsf{c} \neq \mathsf{d} \qquad \mathsf{b} = \mathsf{d} \qquad \mathsf{a} \neq \mathsf{d} \vee \mathsf{a} = \mathsf{c} \qquad \mathsf{a} = \mathsf{b} \vee \mathsf{a} = \mathsf{d} \ .$$

   together with the term order $\mathsf{a} \succ \mathsf{b} \succ \mathsf{c} \succ \mathsf{d}$ giving rise to the following literal order:

$$\mathsf{a} \neq \mathsf{b} \succ_{\mathsf{L}} \mathsf{a} \neq \mathsf{c} \succ_{\mathsf{L}} \mathsf{a} \neq \mathsf{d} \succ_{\mathsf{L}} \mathsf{a} = \mathsf{b} \succ_{\mathsf{L}} \mathsf{a} = \mathsf{c} \succ_{\mathsf{L}} \mathsf{a} = \mathsf{d} \succ_{\mathsf{L}}$$
$$\succ_{\mathsf{L}} \mathsf{b} \neq \mathsf{d} \succ_{\mathsf{L}} \mathsf{b} = \mathsf{c} \succ_{\mathsf{L}} \mathsf{b} = \mathsf{d} \succ_{\mathsf{L}} \mathsf{c} \neq \mathsf{d} \succ_{\mathsf{L}} \mathsf{c} = \mathsf{d} \succ_{\mathsf{L}} \mathsf{d} \neq \mathsf{d} \ .$$

   Prove that $\mathcal{C}$ is unsatisfiable using the superposition calcululs. (20 pts)

5. Determine whether the following statements are true or false. Every correct answer is worth 2 points, every wrong answer -1 points. (22 pts)

Statement	**yes**	**no**
The rules of the DPLL algorithm consist of (i) the tautology rule, (ii) the pure-literal rule, (iii) the two-literals rule, and the (iv) the splitting rule.	☐	☐
Satisfiability of ground Horn clauses is polytime computable.	☐	☐
The collection of all tableau consistent formulas satisfies the satisfaction properties.	☐	☐
Let F be an unsatisfiable sentence and $\mathcal{C}$ its clause form. Then the length of any resolution refutation of $\mathcal{C}$ is polynomially related to the length of the refutation of F in natural deduction.	☐	☐
Structural and strong Skolemisation preserve satisfiability, while optimised Skolemisation preserves validity.	☐	☐
For any resolution method, eliminatation of tautologies during the proof search never affects soundness nor completeness.	☐	☐
Ordered resolution is sound, but requires the addition of functional-reflexive axioms to be complete.	☐	☐
Superposition with equations is sound and complete.	☐	☐
A reductive conditional TRSs is confluent only if all critical pairs converge.	☐	☐
The superposition calculus is slightly overcomplicated. In particular we could drop the 8th inference rule (*equality factoring*), cf. the presentation of the calculus in Exercise 4.	☐	☐
Redundant superposition inferences are liftable.	☐	☐