The formula
This is the proof of the following formula:

$$\neg\exists y(\forall z(\neg P(z,y) \vee \neg\exists x(P(z,x) \wedge P(x,z)) \wedge \exists x(P(z,x) \wedge P(x,z)) \vee P(z,y)))$$

The NNF and Clausal Forms
The skolemized Negation Normal Form of the negated input:

$$\neg A(\beta,a()) \vee \neg A(\beta,\gamma) \vee \neg A(\gamma,\beta) \wedge A(\beta,b(\beta)) \vee A(\beta,a()) \wedge A(b(\beta),\beta) \vee A(\beta,a())$$

The corresponding clausal form:

$$\{\ \neg A(\gamma,\beta), \neg A(\beta,\gamma), \neg A(\beta,a())\ \},\ \{\ A(\beta,a()), A(\beta,b(\beta))\ \},\ \{\ A(b(\beta),\beta), A(\beta,a())\ \}$$

The proof

$$\dfrac{\dfrac{\dfrac{}{\{\ \neg A(\gamma,\beta), \neg A(\beta,\gamma), \neg A(\beta,a())\ \}}\text{ax} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\{\ \neg A(\gamma,\beta), \neg A(\beta,\gamma), \neg A(\beta,a())\ \}}\text{ax} \quad \dfrac{}{\{\ A(\beta,a()), A(\beta,b(\beta))\ \}}\text{ax}}{\{\ A(\beta,b(\beta)), \neg A(\beta,a()), \neg A(a(),\beta)\ \}}\text{res }\sigma_4 \quad \dfrac{}{\{\ A(\beta,a()), A(\beta,b(\beta))\ \}}\text{ax}}{\{\ A(a(),b(a())), \neg A(a(),a())\ \}}\text{res }\sigma_3 \quad \dfrac{}{\{\ A(\beta,a()), A(\beta,b(\beta))\ \}}\text{ax}}{\{\ A(a(),b(a()))\ \}}\text{res }\sigma_2}{}}{\{\ \neg A(b(a()),a())\ \}}\text{res }\sigma_1 \quad \dfrac{\dfrac{\dfrac{}{\{\ \neg A(\gamma,\beta), \neg A(\beta,\gamma), \neg A(\beta,a())\ \}}\text{ax} \quad \dfrac{}{\{\ A(b(\beta),\beta), A(\beta,a())\ \}}\text{ax}}{\{\ A(b(a()),a()), \neg A(a(),a())\ \}}\text{res }\sigma_6 \quad \dfrac{}{\{\ A(b(\beta),\beta), A(\beta,a())\ \}}\text{ax}}{\{\ A(b(a()),a())\ \}}\text{res }\sigma_5}{\Box}\sigma_0$$

The substitutions

$\sigma_0=\{\}$

$\sigma_1=\{\gamma\mapsto a(), \beta\mapsto b(a())\}$

$\sigma_2=\{\beta\mapsto a()\}$

$\sigma_3=\{\beta\mapsto a()\}$

$\sigma_4=\{\gamma\mapsto a()\}$

$\sigma_5=\{\beta\mapsto a()\}$

$\sigma_6=\{\gamma\mapsto a(), \beta\mapsto a()\}$