1. *Solution.*

$$\dfrac{\dfrac{\mathsf{P}(x) \quad \neg\mathsf{P}(x') \vee \mathsf{Q}(\mathsf{f}(x'))}{\mathsf{Q}(\mathsf{f}(x))}\; x' \mapsto x \quad \neg\mathsf{Q}(x'')}{\square}\; x'' \mapsto \mathsf{f}(x)$$

□

2. *Solution.* See Lemma 10.2 and it's proof in the lecture notes. □

3. – *Solution.*

$$A_1 := \forall x \forall y \left(\neg\mathsf{P}(x) \vee \neg\mathsf{P}(y) \vee \mathsf{P}(\mathsf{f}(x,y)) \vee \mathsf{P}(\mathsf{g}(x,y))\right) \ ,$$

   where $\mathsf{f}, \mathsf{g}$ are fresh. □

   – *Solution.* By transformation into PNF and standard Skolemisation we obtain $A_2 := A_1$. □

4. *Solution.*

```
Here is a proof with depth 10, length 15 :
1[0:Inp] || equal(f(f(U)),U)** -> equal(g(U),f(U)).
2[0:Inp] || equal(c,a) -> equal(f(c),c)**.
3[0:Inp] ||  -> equal(b,a)**.
4[0:Inp] ||  -> equal(b,c)**.
5[0:Inp] || equal(g(a),a)** -> .
7[0:SpR:4.0,3.0] ||  -> equal(c,a)**.
11[0:SpR:7.0,2.1] || equal(c,a) -> equal(f(a),a)**.
14[0:SpL:11.1,1.0] || equal(c,a) equal(f(a),a) -> equal(g(a),f(a))**.
18[0:SpL:14.2,5.0] || equal(c,a) equal(f(a),a)** equal(f(a),a)** -> .
22[0:Fac:18.1,18.2] || equal(c,a) equal(f(a),a)** -> .
25[0:SpL:11.1,22.1] || equal(c,a)** equal(c,a)** equal(a,a) -> .
26[0:Fac:25.0,25.1] || equal(c,a)** equal(a,a) -> .
29[0:SpL:7.0,26.0] || equal(a,a)* equal(a,a)* -> .
32[0:EqR:29.0] || equal(a,a)* -> .
36[0:EqR:32.0] ||  -> .
```

□

5.

| **Statement** | **yes** | **no** |
|---|---|---|
| Consider the DPLL rules without splitting rule. These rules give rise to a poly-time decision procedure | ✓ | |
| Satisfiabilty of ground Horn clauses is (at least) NP hard. | | ✓ |
| The semantic tree induced by the clause set $\{\mathsf{P}(x, \mathsf{f}(x)), \neg\mathsf{P}(\mathsf{a}, \mathsf{f}(x)) \vee \mathsf{R}(x), \neg\mathsf{R}(x)\}$ is necessarily open. | | ✓ |
| Consider the collection $S$ of all ground clause sets that are (resolution) consistent. Then $S$ satisifies the satisfaction properties. | ✓ | |
| Let $F$ be an unsatisfiable sentence and $\mathcal{C}$ its clause form. Then the length of any resolution refutation of $\mathcal{C}$ is polynomially related to the length of the refutation of $F$ in natural deduction. | | ✓ |
| Unification is a central ingredient of first-order automated reasoning and can be performed in O(1) time. | | ✓ |
| For any resolution method, eliminatation of tautologies during the proof search never affects soundness nor completeness. | | ✓ |
| Optimised and strong Skolemisation techniques are instances of inner Skolemisation, that is, the place of occurrences of free variables is taken into account. | ✓ | |
| A reductive conditional TRSs is confluent if all critical pairs converge. | ✓ | |
| We can drop the 8th inference rule (*equality factoring*) of the superposition calculus, cf. the presentation of the calculus in Exercise 4, without affecting soundness of the calculus. | ✓ | |