# Functional Programming

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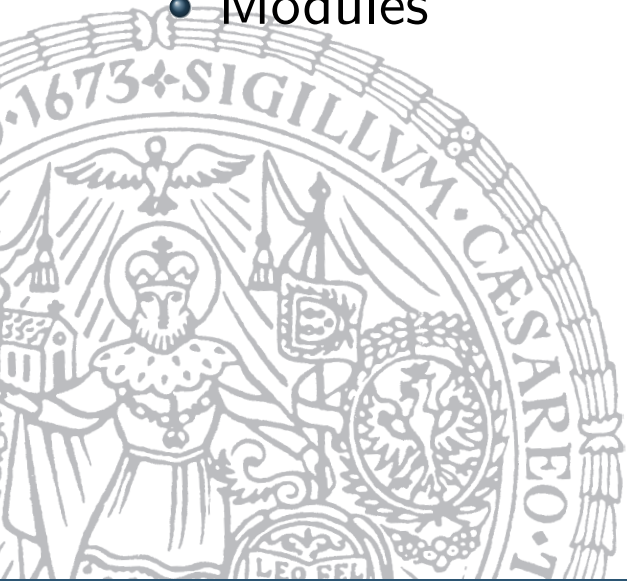
week 2

# Overview

- Week 2 - Lists
  - Summary of Week 1
  - List Basics
  - List Functions
  - Modules



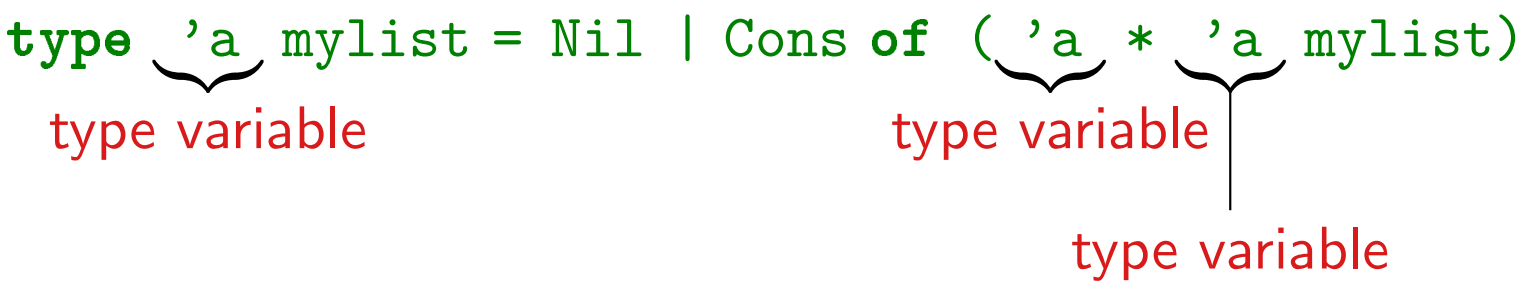
# Overview



# User-defined Types

```
type 'a mylist = Nil | Cons of ('a * 'a mylist)
```

# User-defined Types



# User-defined Types

user-defined type constructor

```
type 'a mylist = Nil | Cons of ('a * 'a mylist)
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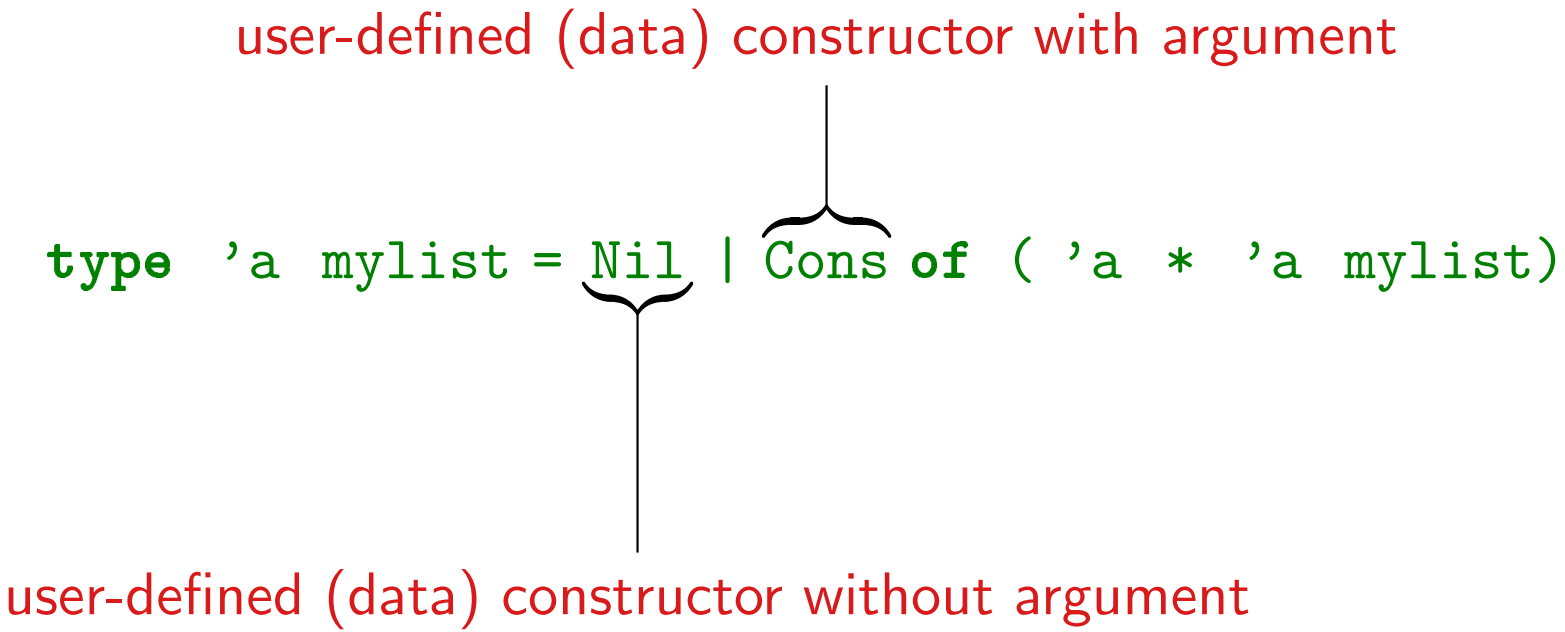
# User-defined Types



# User-defined Types

recursive definition

```
type 'a mylist = Nil | Cons of ('a * 'a mylist)
```

# Recursion, Pattern Matching, and Currying

## Example

```
let rec map (f,ls) = match ls with
  | Nil         -> Nil
  | Cons(x,xs) -> Cons(f x,map(f,xs))
```

# Recursion, Pattern Matching, and Currying (cont'd)

## Currying

```
let rec map f ls = match ls with
  | Nil         -> Nil
  | Cons(x,xs) -> Cons(f x,map f xs)
```

# Recursion, Pattern Matching, and Currying (cont'd)

## Currying

```
let rec map f ls = match ls with
  | Nil         -> Nil
  | Cons(x,xs) -> Cons(f x,map f xs)
```

## Syntactic Sugar

```
let rec map f = function
  | []    -> []
  | x::xs -> f x::map f xs
```

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# This Week

## Practice I

OCaml introduction, lists, strings, trees

## Theory I

lambda-calculus, evaluation strategies, induction,
reasoning about functional programs

## Practice II

efficiency, tail-recursion, combinator-parsing,

## Theory II

type checking, type inference

## Advanced Topics

lazy evaluation, infinite data structures, dependent types, monads

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# The Type of Lists

## Polymorphic Lists

```
type 'a list = [] | (::) of ('a * 'a list)
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# The Type of Lists

## Polymorphic Lists

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type 'a list = [] | (::) of ('a * 'a list)
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predefined type

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type 'a list = [] | (::) of ('a * 'a list)
```

infix 'cons'

# The Type of Lists

## Polymorphic Lists

```
type 'a list = [] | (::) of ('a * 'a list)
```

## Example

| | | | |
|---|---|---|---|
| `[true;false]` | `bool list` | `[(3,2)]` | `(int * int)list` |
| `[1;3;5;7]` | `int list` | `[]` | `'a list` |
| `['a';'b']` | `char list` | `2::3::[]` | `int list` |
| `["str";"w"]` | `string list` | `'a'::['b']` | `char list` |

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# Accessing List Elements - Selectors

```
let hd = function x::_ -> x
                | _    -> failwith "empty list";;

let tl = function _::xs -> xs
                | _     -> failwith "empty list";;
```

# Polymorphic List Functions

## Example (Append)

```
let rec (@) xs ys = match xs with []     -> ys
                                | x::xs -> x::(xs @ ys);;
```

- this function has type `'a list -> 'a list -> 'a list`
- hence it is polymorphic (in `xs` and `ys`)

```
[1;2] @ [3;4]
```

# Polymorphic List Functions

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```
[1;2] @ [3;4]
→  1::([2] @ [3;4])
```

# Polymorphic List Functions

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```
[1;2] @ [3;4]
→  1::([2] @ [3;4])
→  1::2::([] @ [3;4])
```

# Polymorphic List Functions

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[1;2] @ [3;4]
→  1::([2] @ [3;4])
→  1::2::([] @ [3;4])
→  1::2::[3;4]
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# Polymorphic List Functions

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```
[1;2] @ [3;4]
→  1::([2] @ [3;4])
→  1::2::([] @ [3;4])
→  1::2::[3;4]
=  [1;2;3;4]
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
```

- this function has type `int -> 'a -> 'a list`
- hence it is polymorphic (in `x`)

```
replicate 2 'c'
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
```

- this function has type `int -> 'a -> 'a list`
- hence it is polymorphic (in `x`)

```
replicate 2 'c'
 →   if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
```

- this function has type `int -> 'a -> 'a list`
- hence it is polymorphic (in `x`)

```
replicate 2 'c'
→    if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
→+   'c'::replicate 1 'c'
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
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- this function has type `int -> 'a -> 'a list`
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→    if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
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→    'c'::if 1 < 1 then [] else 'c'::replicate (1-1) 'c'
```

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## Example (Replicate)

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let rec replicate n x =
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replicate 2 'c'
 →    if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
 →+   'c'::replicate 1 'c'
 →    'c'::if 1 < 1 then [] else 'c'::replicate (1-1) 'c'
 →+   'c'::'c'::replicate 0 'c'
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
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- this function has type `int -> 'a -> 'a list`
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replicate 2 'c'
 →    if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
 →+   'c'::replicate 1 'c'
 →    'c'::if 1 < 1 then [] else 'c'::replicate (1-1) 'c'
 →+   'c'::'c'::replicate 0 'c'
 →    'c'::'c'::if 0 < 1 then []
                         else 'c'::replicate (0-1) 'c'
```

# Polymorphic List Functions (cont'd)

## Example (Replicate)

```
let rec replicate n x =
  if n < 1 then [] else x::replicate (n-1) x;;
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- this function has type `int -> 'a -> 'a list`
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replicate 2 'c'
 →    if 2 < 1 then [] else 'c'::replicate (2-1) 'c'
 →+   'c'::replicate 1 'c'
 →    'c'::if 1 < 1 then [] else 'c'::replicate (1-1) 'c'
 →+   'c'::'c'::replicate 0 'c'
 →    'c'::'c'::if 0 < 1 then []
                          else 'c'::replicate (0-1) 'c'
 →+   ['c';'c']
```

# Functions on Integer Lists

## Example (Range, Sum, Prod)

```
let rec range m n = if m >= n then []
                                else m::range (m+1) n;;

let rec sum  = function []    -> 0
                      | x::xs -> x + sum xs

let rec prod = function []    -> 1
                      | x::xs -> x * prod xs
```

# Functions on Integer Lists (cont'd)

```
range 1 3 = [1;2]
range 3 2 = []
```

```
sum [1;2;3] = 1 + 2 + 3
    sum [] = 0
```

```
prod [1;2;3] = 1 * 2 * 3
    prod [] = 1
```

$$\texttt{sum(range 1 n)} = \sum_{i=1}^{n-1} i$$

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;

                map succ [1;2;3]
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
 → 2::succ 2::map succ [3]
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
 → 2::succ 2::map succ [3]
 → 2::3::map succ [3]
```

# Higher-Order Functions (map)

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## Example (Map)

```
let rec map f = function      [] -> []
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```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
 → 2::succ 2::map succ [3]
 → 2::3::map succ [3]
 → 2::3::succ 3::map succ []
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
 → 2::succ 2::map succ [3]
 → 2::3::map succ [3]
 → 2::3::succ 3::map succ []
 → 2::3::4::map succ []
```

# Higher-Order Functions (map)

- functions taking functions as arguments

## Example (Map)

```
let rec map f = function      [] -> []
                        | x::xs -> f x::map f xs;;
```

```
map succ [1;2;3]
 → succ 1::map succ [2;3]
 → 2::map succ [2;3]
 → 2::succ 2::map succ [3]
 → 2::3::map succ [3]
 → 2::3::succ 3::map succ []
 → 2::3::4::map succ []
 → 2::3::4::[] = [2;3;4]
```

# Higher-Order Functions (foldr)

```
let rec foldr f b = function      [] -> b
                          | x::xs -> f x (foldr f b xs);;
```

- this function has type
  `('a -> 'b -> 'b) -> 'b -> 'a list -> 'b`
- this function computes

  `foldr f b [e1;e2;...;eN]` = `f e1 (f e2 (...(f eN b)...))`

- this function is useful to implement other functions

```
let sum  xs = Lst.foldr ( + ) 0 xs;;

let prod xs = Lst.foldr ( * ) 1 xs;;
```

- here `Lst` refers to a module (see next section)

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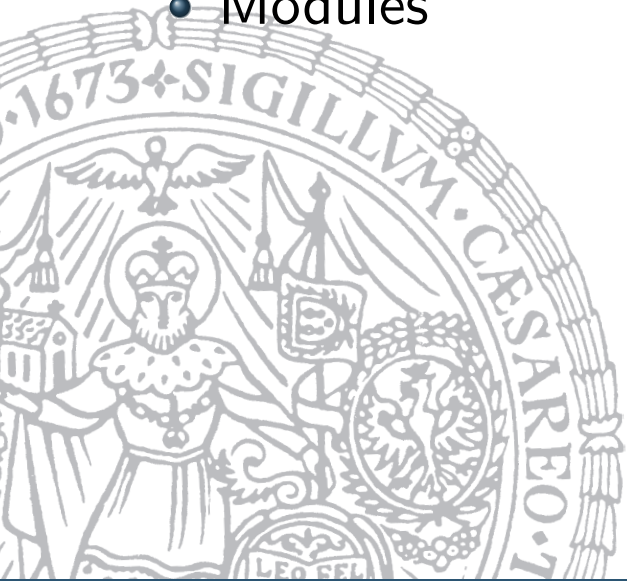

# Structuring Code

## Modules are used to ...

- split source code into several files
- separate namespaces for functions and types
- abstract from concrete representations

# Module Basics - Split Source Code

- for each module *Module* create implementation file *module*.ml
- code of each module goes into corresponding *.ml file

## Example

```
let hd = ...
let tl = ...
let rec map f = ...
let rec foldr f b = ...
```

Lst.ml

```
let rec range m n = ...
let sum = ...
let prod = ...
```

IntLst.ml

# Module Basics - Separate Namespaces

- refer to function *fun* from module *Module* by *Module*.*fun*
- no problem to have same function names in different modules

## Example

Compute greatest number that can be encoded in binary using *n* bits

```
let pow2 n = IntLst.prod(Lst.replicate n 2);;
```

Int.ml

```
let main() =
 let n = read_int() in
 let r =
  IntLst.sum (Lst.map Int.pow2 (IntLst.range 0 n)) in
 Printf.printf "%i\n" r
in main()
```

maxbin.ml

# Module Basics - Abstraction

- create interface file *module*`.mli` for module *Module*
- signature (i.e., names and types) of module goes into corresponding `*.mli` file

## Example

```
type 'a t
val empty : 'a t
val push : 'a -> 'a t -> 'a t
val pop : 'a t -> ('a * 'a t)
```

stck.mli

```
open Lst
type 'a t = 'a list
let empty = []
let push e s = e::s
let pop s = (hd s,tl s)
```

stck.ml

```
type 'a t =
  | Empty
  | Full of ('a * 'a t)
...
```

stck.ml