Functional Programming

WS 2015/16

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week 04

Overview

- Week 4 - Trees
    - Summary of Week 3
    - Rooted Trees
    - Binary Trees
    - Huffman Coding

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L-Strings

- strings not functional in OCaml
- therefore use module Strng

L-Strings as character lists

```
type t = char list
val of_string : string -> char list
val to_string : char list -> string
val of_int : int -> char list
val print : char list -> unit
val toplevel_printer : Format.formatter -> char list -> unit
val blanks : int -> t
```

Setting Up the Interpreter

- `.ocamlinit` (searched in `.` and `~`)
- write modules for custom interpreter to *file*`.mltop`
- compile with '`ocamlbuild` *file*`.top`'
- start with '`./`*file*`.top`'

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Example

```
AsciiArt
Lst
Picture
Strng
```

w03.mltop

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This Week

Practice I

OCaml introduction, lists, strings, trees

Theory I

lambda-calculus, evaluation strategies, induction,
reasoning about functional programs

Practice II

efficiency, tail-recursion, combinator-parsing,

Theory II

type checking, type inference

Advanced Topics

lazy evaluation, infinite data structures, dependent types, monads

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What Are Trees?

Definition (Tree)

(rooted) tree $T = (N, E)$

- set of nodes N
- set of edges $E \subseteq N \times N$
- unique root of T ($root(T) \in N$) without predecessor
- all other nodes have exactly one predecessor
- leaf is node without successor

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What Are Trees? (cont'd)

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- $N = \{a, b, c, d, e, f, g\}$
- $E = \{(a, b), (a, c), (a, e), (c, d), (e, f), (e, g)\}$
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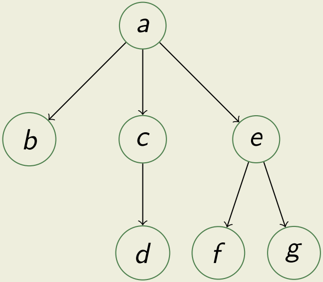
- $T =$



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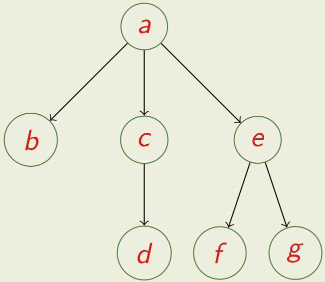

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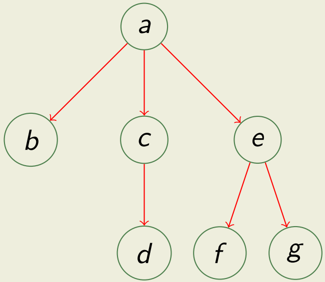

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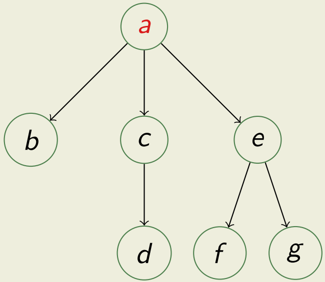

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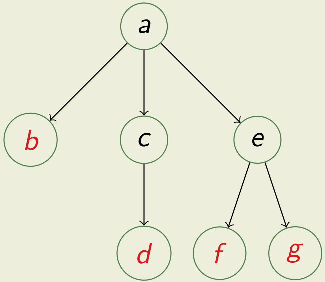

Trees in OCaml

Type

```
type 'a tree = Empty | Node of 'a * 'a tree list
```

Trees in OCaml

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empty tree (annotating `Empty`)

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Trees in OCaml

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```

node with content (annotation under `Node`)

Trees in OCaml

Type

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```

Example

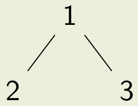
`Empty`

1

`Node(1,[Empty])`

1

`Node(1,[])`



`Node(1,[Node(2,[]);Node(3,[])])`

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Restricting the Branching-Factor

Definition (Binary tree)

restrict number of successors (maximal 2)

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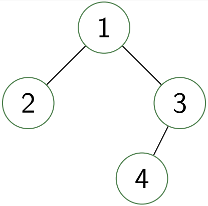


```
Node(Node(Empty,2,Empty),
     1,
     Node(Node(Empty,4,Empty),3,Empty))
```

Functions on BinTrees

Definition (Size)

size of a tree equals number of nodes

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let rec size = function
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height of a tree is length of longest path from root to some leaf

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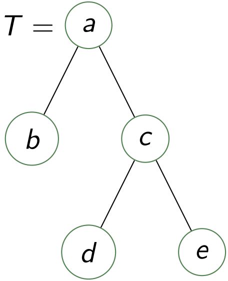
Definition (Height)

height of a tree is length of longest path from root to some leaf

```
let rec height = function
  | Empty        -> 0
  | Node(l,_,r) -> max (height l) (height r) + 1
```

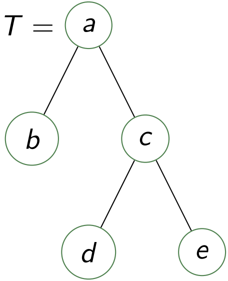
Example

- convention: do not draw 'Empty' nodes
- `size` $T = ?$
- `height` $T = ?$



Example

- convention: do not draw 'Empty' nodes
- `size` $T = 5$
- `height` $T = ?$



Example

- convention: do not draw ‘Empty’ nodes
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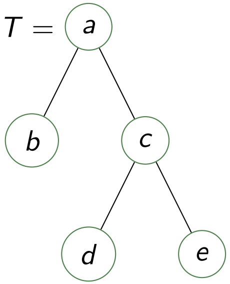
- `height` $T = 3$



Creating Trees of Lists

The easy way

```
let rec of_list = function []     -> Empty
                         | x::xs -> Node(Empty,x,of_list xs)
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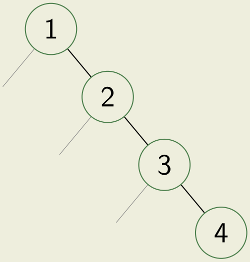

Creating Trees of Lists (cont'd)

The fair way

```
let rec make = function
 | [] -> Empty
 | xs ->
  let m = Lst.length xs / 2 in
  let (ys,zs) = Lst.split_at m xs in
  Node (make ys,Lst.hd zs,make(Lst.tl zs))
```

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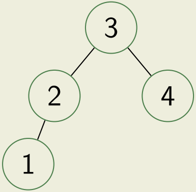

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Ordered insertion

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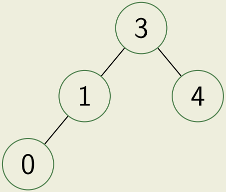
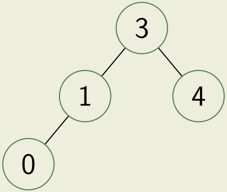
Example

`insert compare 2` 

$\to^+$ 

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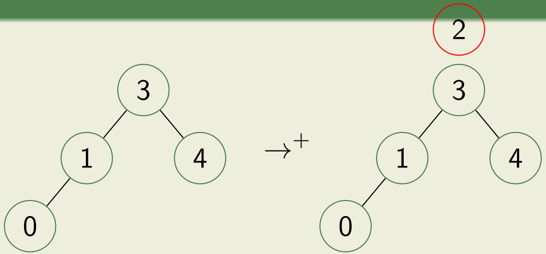

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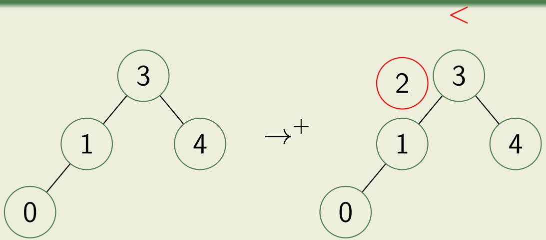

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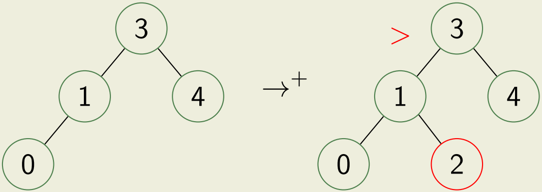

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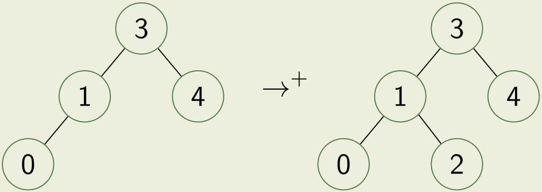

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Search trees

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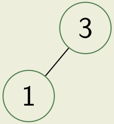

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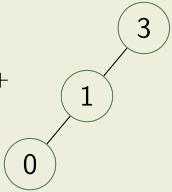

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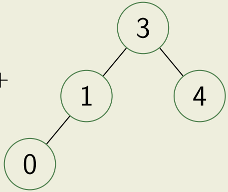

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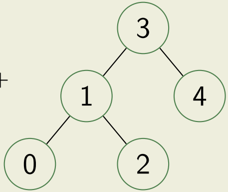

Transforming Trees Into Lists

Flatten

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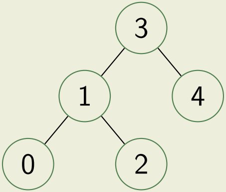
`flatten`  $\to^+$

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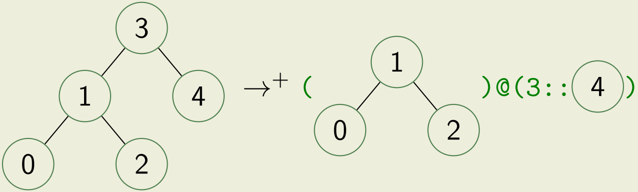

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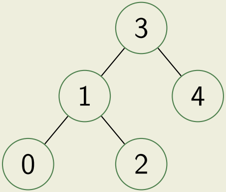

$\to^{+}$ `((`0`)@(1::`2`))@(3::`4`)`

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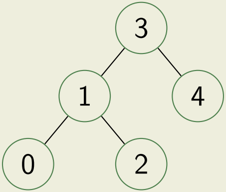
`flatten`  $\to^+$ `(([0])@(1::[2]))@(3::[4])`

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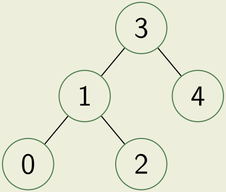
flatten  $\to^+$ [0;1;2;3;4]

A Sorting Algorithm for Lists

```
let sort c xs = BinTree.flatten(BinTree.search_tree c xs)
```

Overview

The Idea

Reduce storage size

- `ASCII` uses 1 byte per character
- encode frequent characters 'short'

Example

Text: '`text`'

- 32 bits in `ASCII` (01110100011001010111100001110100)
- using

t	$\mapsto$	0
e	$\mapsto$	10
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6 bits needed (010110)

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- encode frequent characters 'short'

Example

Text: 'text'

- 32 bits in ASCII (01110100011001010111100001110100)
- using

t	$\mapsto$	0
e	$\mapsto$	10
x	$\mapsto$	11

6 bits needed (010110)

The Idea

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Some More Useful List Functions

```
let concat xs = foldr (@) [] xs

let rec take_while p = function
 | []    -> []
 | x::xs -> if p x then x :: take_while p xs else []

let rec drop_while p = function
 | []               -> []
 | x::xs as list -> if p x then drop_while p xs else list

let span p xs = (take_while p xs, drop_while p xs)

let rec until p f x = if p x then x else until p f (f x)
```

Counting Symbol Frequency

Collate

```
let rec collate = function
 | []           -> []
 | w::ws as xs ->
  let (ys,zs) = Lst.span ((=)w) xs in
  (Lst.length ys,w) :: collate zs
```

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Example

collate ['a';'a';'b';'c';'c';'c'] $\to^{+}$

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Example

collate ['a';'a';'b';'c';'c';'c'] $\to^+$
[(2,'a');(1,'b');(3,'c')]

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Example

collate ['a';'a';'b';'c';'c';'c'] $\to^+$
  [(2,'a');(1,'b');(3,'c')]

collate ['a';'a';'b';'a';'a';'a'] $\to^+$

Counting Symbol Frequency

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Example

collate ['a';'a';'b';'c';'c';'c'] $\rightarrow^+$
[(2,'a');(1,'b');(3,'c')]

collate ['a';'a';'b';'a';'a';'a'] $\rightarrow^+$
[(2,'a');(1,'b');(3,'a')]

Generating a Symbol-Frequency List

Sample

```
let sample xs = sort compare (collate(sort compare xs))
```

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let sample xs = sort compare (collate(sort compare xs))
```

Example

sample ['t';'e';'x';'t'] $\to^+$

Generating a Symbol-Frequency List

Sample

```
let sample xs = sort compare (collate(sort compare xs))
```

Example

sample ['t';'e';'x';'t'] $\to^+$ [(1,'e');(1,'x');(2,'t')]

Huffman Trees

- leaf nodes contain weight (= frequency) + character
- other nodes store sum of weights of subtrees

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`type 'a option = None | Some of 'a` (predefined)

```
type node = (int * char option)

type t = node btree
```

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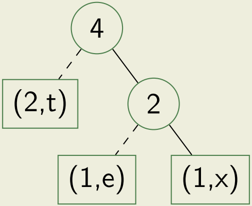
Example



Building the Huffman Tree

Step 1

- transform the symbol-frequency list into a list of Huffman trees

```
let mknode (w,c) = Node(Empty,(w,Some c),Empty)
```

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Example

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Lst.map mknode [(1,'e');(1,'x');(2,'t')]
```

$\to^{+}$

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- transform the symbol-frequency list into a list of Huffman trees

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let mknode (w,c) = Node(Empty,(w,Some c),Empty)
```

Example

```
Lst.map mknode [(1,'e');(1,'x');(2,'t')]
```

$\to^+$ [$\boxed{(1, e)}$; $\boxed{(1, x)}$; $\boxed{(2, t)}$]

Building the Huffman Tree (cont'd)

Step 2

- combine first two trees until only one left

```
let weight = function
 | Node(_,(w,_),_) -> w
 | _                -> failwith "empty␣tree"

let combine = function
 | xt::yt::xts -> let w = weight xt + weight yt in
  insert (Node(xt,(w,None),yt)) xts
 | _            -> failwith "length␣has␣to␣be␣greater␣than␣1"
let is_singleton x = Lst.length x = 1

let insert vt wts =
 let (xts,yts) =
  Lst.span (fun x -> weight x <= weight vt) wts in
 xts@(vt::yts)
```

Building the Huffman Tree (cont'd)

Step 2 (cont'd)

- combine first two trees until only one left

```
let tree xs =
 let ts = Lst.map mknode (sample xs) in
 Lst.hd (Lst.until is_singleton combine ts)
```

Building the Huffman Tree (cont'd)

Step 2 (cont'd)

- combine first two trees until only one left

```
let tree xs =
 let ts = Lst.map mknode (sample xs) in
 Lst.hd (Lst.until is_singleton combine ts)
```

Example

tree ['t';'e';'x';'t'] $\to^{+}$

Building the Huffman Tree (cont'd)

Step 2 (cont'd)

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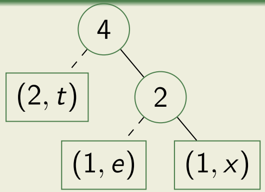
Example

`tree ['t';'e';'x';'t']` $\to^+$



Generating a Code-Table

Encoding

- Which code corresponds to a given character?

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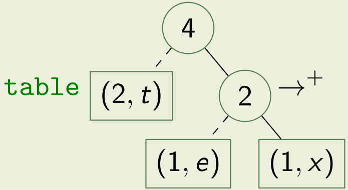
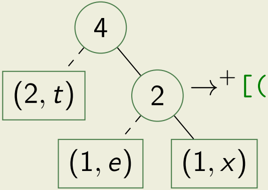


Generating a Code-Table

Encoding

- Which code corresponds to a given character?

Example

`table` 

$\to^+$ `[('t',[0]);('e',[1;0]);('x',[1;1])]`

Generating a Code-Table (cont'd)

Encoding

- Which code corresponds to a given character?

```
let table t =
 let rec tab code = function
  | Node(Empty,(_,Some v),Empty) -> [(v,code)]
  | Node(l,_,r) -> tab (code@[0]) l @ tab (code@[1]) r
  | _ -> failwith "the␣Huffman␣tree␣is␣empty"
 in tab [] t
```

Encoding

- use code-table for compression

```
let encode t text =
 let tab = table t in
 Lst.concat(Lst.map (lookup tab) text)

let rec lookup tab c = match tab with
 | ((v,code)::tab) -> if v = c then code else lookup tab c
 | _               -> failwith "not␣found"
```

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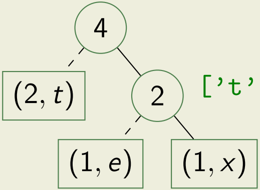
encode 

['t';'e';'x';'t'] $\to^+$

Encoding

- use code-table for compression

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let encode t text =
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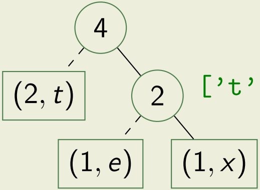
Example

encode  ['t';'e';'x';'t'] $\to^+$ [0;1;0;1;1;0]

Decoding

- use Huffman tree for decompression

```
let rec decode_char = function
 | (Node(Empty,(_,Some c),Empty),cs) -> (c,cs)
 | (Node(xt,_,_),0::cs)              -> decode_char (xt,cs)
 | (Node(_,_,xt),1::cs)              -> decode_char (xt,cs)
 | _                                 -> failwith "empty␣tree"

let rec decode t = function
 | [] -> []
 | xs -> let (c,xs) = decode_char (t,xs) in c::decode t xs
```