

Logic Programming

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Summary of Last Lecture

Definitions

- a **type** is a (possible infinite) set of terms
- types are conveniently defined by unary relations
- a type is **complete** if closed under the instance relation
- with every complete type T one associates an **incomplete** type IT which is a set of terms with instances in T and instances not in T

Example (composition of programs)

```
ancestor_of_2(Ancessor, Predecessor) :-  
    child_of(Predecessor, Ancessor),  
ancestor_of_2(Ancessor, Predecessor) :-  
    ancestor_of_2(Person, Predecessor),  
    child_of(Person, Ancessor).
```

Termination Analysis

Fact

- *for termination analysis only recursive calls (cycles in call tree) are essential*
- *let's remove non-recursive rules*

Example (specialised)

```

ancestor_of_2(Ancestor, Predecessor) :- false,
    child_of(Predecessor, Ancestor),
ancestor_of_2(Ancestor, Predecessor) :-
    ancestor_of_2(Person, Predecessor),
    child_of(Person, Ancestor).

```

equivalently

```

ancestor_of_2(Ancestor, Predecessor) :-
    ancestor_of_2(Person, Predecessor),
    child_of(Person, Ancestor).

```

Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, unification, database and recursive programming, termination

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

Full Prolog

semantics, correctness proofs, meta-logical predicates, cuts nondeterministic programming, efficient programs, complexity

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Composition of Programs

five steps to implement relation R

- 1 look up existing definitions of relation R

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 - family relations
 - train tables
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Composition of Programs

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- 1 look up existing definitions of relation R
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 - mainly for documentation
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- 1 look up existing definitions of relation R
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- 3 think up a suitable name
 - convert a verbose description into a name
 - `child_of`
- 4 write assertions (use cases)
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Negative Definitions

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Example

```
← land(27).  
← land(kar_rinne).  
← land(milka_kuh).
```

Structured Data and Data Abstraction

Example (Unstructured Data)

```
course(discrete_mathematics,tuesday,8,11,sandor,szedmak,  
victor_franz_hess,d).
```

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course(discrete_mathematics,time(tuesday,8,11),  
lecturer(sandor,szedmak),location(victor_franz_hess,d)).
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Structured Data and Data Abstraction

Example (Unstructured Data)

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```

Example

```
lecturer(Lecturer,Course) :-
  course(Course,Time,Lecturer,Location).

duration(Course,Length) :-
  course(Course,time(Day,Start,Finish),Lecturer,Location),
  plus(Start,Length,Finish).
```

Example (cont'd)

```
teaches(Lecturer,Day) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location).
```

```
occupied(Location,Day,Time) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location),  
    Start ≤ Time, Time ≤ Finish.
```

Example (cont'd)

```
teaches(Lecturer,Day) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location).
```

```
occupied(Location,Day,Time) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location),  
    Start <= Time, Time <= Finish.
```

Why structure Data?

- helps to organise data
- rules can be written abstractly, hiding irrelevant detail
- modularity is improved

Example (cont'd)

```
teaches(Lecturer,Day) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location).
```

```
occupied(Location,Day,Time) :-  
    course(Course,time(Day,Start,Finish),Lecturer,Location),  
    Start <= Time, Time <= Finish.
```

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- helps to organise data
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- modularity is improved

The Art of Prolog says

We believe that the appearance of a program is important, particularly when attempting difficult problems

Logic Programs and the Relational Database Model

Observation

the basic operations of relational algebras, namely:

- 1 union
- 2 difference
- 3 cartesian product
- 4 projection
- 5 selection
- 6 intersection

can easily be expressed within logic programming

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Example

```
r_union_s( $X_1, \dots, X_n$ ) :- r( $X_1, \dots, X_n$ ).
```

```
r_union_s( $X_1, \dots, X_n$ ) :- s( $X_1, \dots, X_n$ ).
```

Example (Uses of append)

```
prefix(Xs,Ys) :- append(Xs,As,Ys).
```

```
suffix(Xs,Ys) :- append(As,Xs,Ys).
```

```
member(X,Ys) :- append(As,[X|Xs],Ys).
```

Example (Uses of append)

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prefix(Xs,Ys) :- append(Xs,As,Ys).  
suffix(Xs,Ys) :- append(As,Xs,Ys).  
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```

Example

```
reverse([],[]).  
reverse([X|Xs],Zs) :- reverse(Xs,Ys), append(Ys,[X],Zs).
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Example

```
reverse([],[]).  
reverse([X|Xs],Zs) :- reverse(Xs,Ys), append(Ys,[X],Zs).  
reverse(Xs,Ys) :- reverse(Xs,[],Ys).  
reverse([X|Xs],Acc,Ys) :- reverse(Xs,[X|Acc],Ys).  
reverse([],Ys,Ys).
```

Example (Uses of append)

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prefix(Xs,Ys) :- append(Xs,As,Ys).
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```

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reverse([X|Xs],Acc,Ys) :- reverse(Xs,[X|Acc],Ys).
reverse([],Ys,Ys).

```

Example

```

length([],0).
length([X|Xs],s(N)) :- length(Xs,N).

```

Example

```
select(X, [X|Xs], Xs).  
select(X, [Y|Ys], [Y|Zs]) :- select(X, Ys, Zs).
```

Example

```
permutation_sort(Xs,Ys) :- permutation(Xs,Ys), ordered(Ys).
```

```
select(X, [X|Xs], Xs).
```

```
select(X, [Y|Ys], [Y|Zs]) :- select(X,Ys,Zs).
```

Example

```
permutationsort(Xs,Ys) :- permutation(Xs,Ys), ordered(Ys).  
permutation(Xs,[Z|Zs]) :- select(Z,Xs,Ys), permutation(Ys,Zs).  
permutation([],[]).
```

```
select(X,[X|Xs],Xs).  
select(X,[Y|Ys],[Y|Zs]) :- select(X,Ys,Zs).
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Example

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permutation_sort(Xs,Ys) :- permutation(Xs,Ys), ordered(Ys).  
permutation(Xs,[Z|Zs]) :- select(Z,Xs,Ys), permutation(Ys,Zs).  
permutation([],[]).  
ordered([X]).  
ordered([X,Y|Ys]) :- X <= Y, ordered([Y|Ys]).  
select(X,[X|Xs],Xs).  
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```

Example

```

insertionsort([X|Xs],Ys) :- insertionsort(Xs,Zs),
                             insert(X,Zs,Ys).
insertionsort([],[]).

```

Example

```

permutation_sort(Xs,Ys) :- permutation(Xs,Ys), ordered(Ys).
permutation(Xs,[Z|Zs]) :- select(Z,Xs,Ys), permutation(Ys,Zs).
permutation([],[]).
ordered([X]).
ordered([X,Y|Ys]) :- X <= Y, ordered([Y|Ys]).
select(X,[X|Xs],Xs).
select(X,[Y|Ys],[Y|Zs]) :- select(X,Ys,Zs).

```

Example

```

insertion_sort([X|Xs],Ys) :- insertion_sort(Xs,Zs),
                             insert(X,Zs,Ys).
insertion_sort([],[]).
insert(X,[],[X]).
insert(X,[Y|Ys],[Y|Zs]) :- X > Y, insert(X,Ys,Zs).
insert(X,[Y|Ys],[X,Y|Ys]) :- X <= Y.

```

Example

```
quicksort([X|Xs],Ys) :-  
    partition(Xs,X,Littles,Bigs),  
    quicksort(Littles,Ls), quicksort(Bigs,Rs),  
    append(Ls,[X|Rs],Ys).
```

Example

```
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    quicksort(Littles,Ls), quicksort(Bigs,Rs),  
    append(Ls,[X|Rs],Ys).
```

```
partition([X|Xs],Y,[X|Ls],Bs) :-  
    X <= Y, partition(Xs,Y,Ls,Bs).  
partition([X|Xs],Y,Ls,[X|Bs]) :-  
    X > Y, partition(Xs,Y,Ls,Bs).  
partition([],Y,[],[]).
```

Example

```
quicksort([X|Xs],Ys) :-
    partition(Xs,X,Littles,Bigs),
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partition([X|Xs],Y,[X|Ls],Bs) :-
    X <= Y, partition(Xs,Y,Ls,Bs).
partition([X|Xs],Y,Ls,[X|Bs]) :-
    X > Y, partition(Xs,Y,Ls,Bs).
partition([],Y,[],[]).
```

Example (Recursive Datastructures)

```
isotree(nil,nil).
isotree(tree(X,Left1,Right1),tree(X,Left2,Right2)) :-
    isotree(Left1,Left2), isotree(Right1,Right2).
isotree(tree(X,Left1,Right1),tree(X,Left2,Right2)) :-
    isotree(Left1,Right2), isotree(Right1,Left2).
```

Accessing compound terms

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Example

```
: - functor(father(haran,lot),F,A)
```

```
F ↦ father
```

```
A ↦ 2
```

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Example

```
: - functor(father(haran,lot),F,A)
```

```
F ↦ father
```

```
A ↦ 2
```

Example

```
: - arg(2,father(haran,lot),X)
```

```
X ↦ lot
```

Example

```
subterm(Term,Term).
subterm(Sub,Term) :-
    compound(Term),
    functor(Term,F,N),
    subterm(N,Sub,Term).

subterm(N,Sub,Term) :-
    N > 1,
    N1 is N - 1,
    subterm(N1,Sub,Term).

subterm(N,Sub,Term) :-
    arg(N,Term,Arg),
    subterm(Sub,Arg).
```

Definition

- $Term =.. List$ is true if $List$ is a list whose head is the principal functor of $Term$, and whose tail is the list of arguments of $Term$
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: - father(haran,lot) =.. Xs
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X ↦ [father,haran,lot]
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- programs written with `functor` and `arg` can also be written with `univ`
- programs using `univ` are typically simpler
- programs using `functor` and `arg` are more efficient
- `univ` can be built from `functor` and `arg`

Approach

- 1 sometimes it is useful (easier) to think of a relation as a function
- 2 use this definition for coding
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delete([X|Xs],Z,?)      :- X = Z                .
delete([X|Xs],Z,?)      :- dif(X,Z),            .
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delete([],X,[]).
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delete([X|Xs],X,Ys) :- delete(Xs,X,Ys).  
delete([X|Xs],Z,[X|Ys]) :- delete(Xs,Z,Ys).  
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```

Example

```
delete2([X|Xs],X,Ys) :- delete2(Xs,X,Ys).  
delete2([X|Xs],Z,[X|Ys]) :- delete2(Xs,Z,Ys).  
delete2([],X,[]).
```

Example

```
delete2([X|Xs],X,Ys) :- delete2(Xs,X,Ys).  
delete2([X|Xs],Z,[X|Ys]) :- delete2(Xs,Z,Ys).  
delete2([],X,[]).  
  
:- delete2([a,b,c,b],b,[a,c])  
true  
  
:- delete2([a,b,c,b],b,[a,b,c,b])  
true
```

Example

```

delete2([X|Xs],X,Ys) :- delete2(Xs,X,Ys).
delete2([X|Xs],Z,[X|Ys]) :- delete2(Xs,Z,Ys).
delete2([],X,[]).

:- delete2([a,b,c,b],b,[a,c])
true

:- delete2([a,b,c,b],b,[a,b,c,b])
true

```

Example

```

select(X,[X|Xs],Xs).
select(X,[Y|Ys],[Y|Zs]) :- select(X,Ys,Zs)

:- delete([a],b,[a])
true

:- select([a],b,X)
false

```

GUPU

Example (termination)

- Example 29
- Example 30

GUPU

Example (termination)

- Example 29
- Example 30

Example (definite clause grammars)

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