Computational Logic

# Logic Programming

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# Summary of Last Lecture

## Definition

given a list `[1,2,3]` it can be represented as the difference of two lists; the difference of two lists is denotes as $As \setminus Bs$ and called difference list

## Example

```
expr(knoten(1,[])) -->
        "1".
expr(knoten(+,[Expr1,Expr2])) -->
        "(",
        expr(Expr1),
        "+",
        expr(Expr2),
        ")".
```

# Outline of the Lecture

## Monotone Logic Programs

introduction, basic constructs, unification, database and recursive programming, termination

## Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

## Full Prolog

semantics, correctness proofs, meta-logical predicates, cuts nondeterministic programming, efficient programs, complexity

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# Difference-structures

## Example

convert the sum $(a + b) + (c + d)$ into $(a + (b + (c + (d + 0))))$

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## Example

```
normalise(Exp,Norm) :- normalise_ds(Exp,Norm ++ 0).

normalise_ds(A+B, Norm ++ Norm0) :-
    normalise_ds(A, Norm ++ NormB),
    normalise_ds(B, NormB ++ Norm0).
normalise_ds(A,(A + Norm) ++ Norm) :-
    constant(A).
```

## Example (Definite Clause Grammars)

```
sentence(sentence(N,V)) → noun_phrase(N), verb_phrase(V).

noun_phrase(np(D,N)) → determiner(D), noun_phrase2(N).
noun_phrase(np(N)) → noun_phrase2(N).

noun_phrase2(np2(A,N)) → adjective(A), noun_phrase2(N).
noun_phrase2(np2(N)) → noun(N).

verb_phrase(vp(V,N)) → verb(V), noun_phrase(N).
verb_phrase(vp(V)) → verb(V). determiner → [the].
determiner → [a].

noun → [pie-plate].
noun → [surprise].

adjective → [decorated].

verb → [contains].
```

sentence(PT) $\stackrel{*}{\Rightarrow}$ "the decorated pie-plate contains a surprise"

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## Example

```
determiner(det(the)) → [the].
determiner(det(a)) → [a].

noun(noun(pie-plate)) → [pie-plate].
noun(noun(pie-plates)) → [pie-plates].
noun(noun(surprise)) → [surprise].
noun(noun(surprises)) → [surprises].

adjective(adj(decorated)) → [decorated].

verb(verb(contains)) → [contains].
verb(verb(contain)) → [contain].
```

sentence(PT) $\stackrel{*}{\Rightarrow}$ ‘‘the decorated pie-plates contains a surprise’’

# Extension: Number Agreement

## Example

```
sentence(sentence(NP,VP),Num) →
    noun_phrase(N,Num), verb_phrase(V,Num).

⋮

determiner(det(the),Num) → [the].
determiner(det(a),singular) → [a].

noun(noun(pie-plate),singular) → [pie-plate].
noun(noun(pie-plates),plural) → [pie-plates].
noun(noun(surprise),singular) → [surprise].
noun(noun(surprises),plural) → [surprises].

adjective(adj(decorated)) → [decorated].

verb(verb(contains),singular) → [contains].
verb(verb(contain),plural) → [contain].
```

$\texttt{sentence(PT)} \overset{*}{\Rightarrow}$ ‘‘the decorated pie-plates contain a surprise’’

## Example

```
sentence -->
    subject,
    predicate.

subject -->
    [the], [big], [bear].
subject -->
    "the", "little", "lion".

predicate -->
    [roars].
predicate -->
    [is, happy].
predicate -->
    [lives, in, the, golden, city].
```

## Example

```
sentence -->
    subject,
    predicate.

subject -->
    [the], [big], [bear].
subject -->
    "the", "little", "lion".

predicate -->
    [roars].
predicate -->
    [is, happy].
predicate -->
    [lives, in, the, golden, city].

:- phrase(sentence,Text), Text = [the, big, bear, roars].
:- phrase(sentence,Text), Text = [116, 104, 101, 108, 105|_].
```

# Regular Predicate from Within a DCG

## Task

write a DCG for number(N) that recognised numbers in English:

```
?- phrase(number(N),"onehundredandseventyfive").
N = 175 ;
```

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Prolog provides an arithmetical interface

*Value* `is` *Expression*

# Regular Predicate from Within a DCG

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N = 175 ;
```

## Definition

Prolog provides an arithmetical interface

*Value* `is` *Expression*

## Example

| | | |
|---|---|---|
| `X is 3+5` | `8 is 3+5` | `N is N+1` |
| `X` $\mapsto$ `8` | `true` | *`nonsensical`* |

## Solution

```
number(0) --> "zero".
number(N) --> xxx(N).

xxx(N) --> digit(D), "hundred", rest_xxx(N1),
           {N is D * 100 + N1}.

rest_xxx(0) --> "".
rest_xxx(N) --> "and", xx(N).

xx(N) --> digit(N).
xx(N) --> teen(N).
xx(N) --> tens(T), rest_xx(N1), {N is T + N1}.

rest_xx(0) --> "".
rest_xx(N) --> digit(N).

digit(1) --> "one".
digit(2) --> "two".
teen(10) --> "ten".
tens(20) --> "twenty".
```

# Generate and Test

## Theorem (Four Colour Theorem)

*no more than four colours are required to colour the regions of a map so that no two adjacent regions have the same color*

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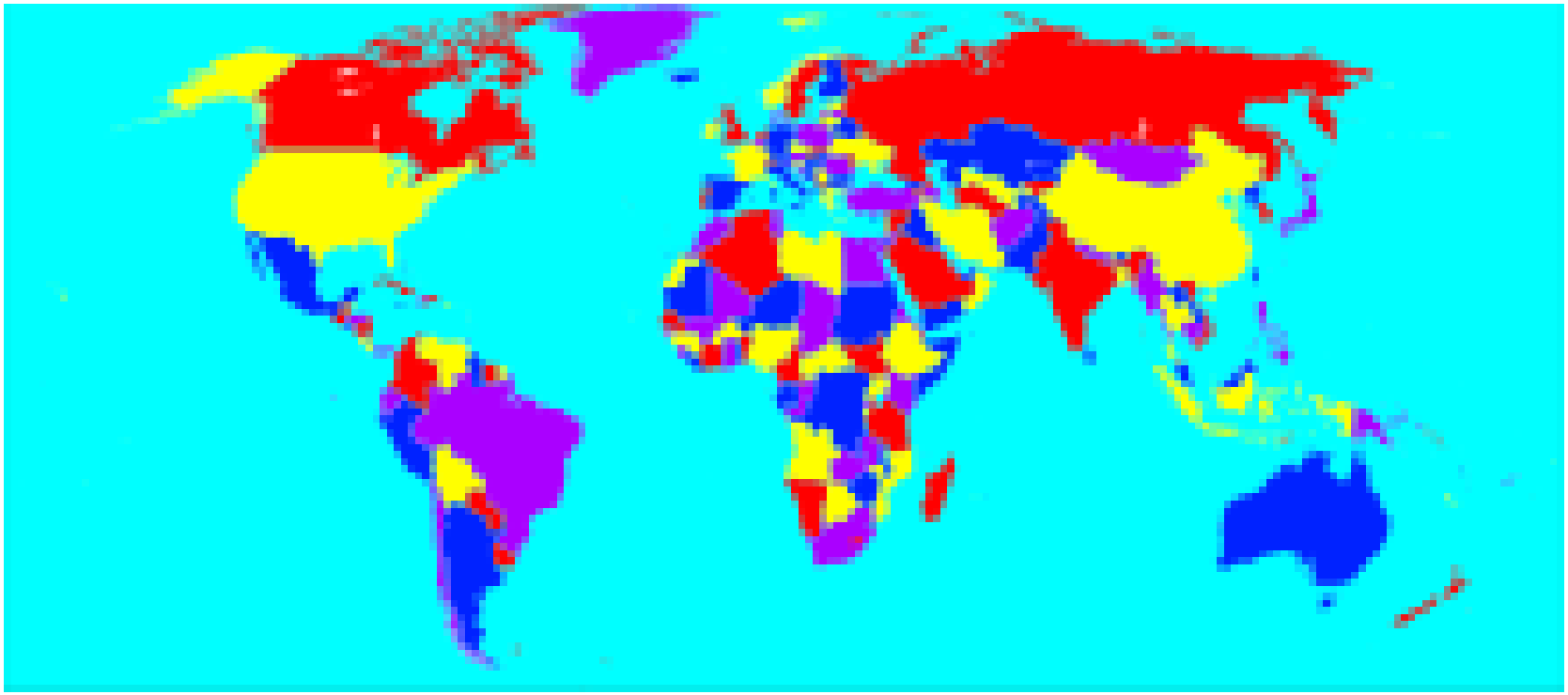
## Example



# Auxiliary Predicates

## Example

```
select(X,[X|Xs],Xs).
select(X,[Y|Ys],[Y|Zs]) :-
    select(X,Ys,Zs).

member(X,[X|_Xs]).
member(X,[_Y|Xs]) :-
    member(X,Xs).

:- sublist_of([b,d],[a,b,c,d]).

sublist_of([],_Ys).
sublist_of([X|Xs],Ys) :-
    member(X,Ys),
    sublist_of(Xs,Ys).
```

# Generate and Test

## Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
    region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
    region(e,E,[B,C,F]), region(f,F,[C,D,E])]).
```

# Generate and Test

## Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
    region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
    region(e,E,[B,C,F]), region(f,F,[C,D,E])]).

coloured_map([Region|Regions], Colours) :-
    coloured_region(Region,Colours),
    coloured_map(Regions,Colours).
coloured_map([],Colours).
```

# Generate and Test

## Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
    region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
    region(e,E,[B,C,F]), region(f,F,[C,D,E])]).

coloured_map([Region|Regions], Colours) :-
    coloured_region(Region,Colours),
    coloured_map(Regions,Colours).
coloured_map([],Colours).

coloured_region(region(Name,Colour,Neighbours), Colours) :-
    select(Colour,Colours,Colours1),
    sublist_of(Neighbours,Colours1).
```

# Generate and Test

## Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
    region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
    region(e,E,[B,C,F]), region(f,F,[C,D,E])]).

coloured_map([Region|Regions], Colours) :-
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coloured_map([],Colours).

coloured_region(region(Name,Colour,Neighbours), Colours) :-
    select(Colour,Colours,Colours1),
    sublist_of(Neighbours,Colours1).

test_colour(Map) :-
    is_map(Map),
    is_colours(Colours),
    coloured_map(Map,Colours).
```

# Nondeterministic Programming

## Example

| | 0 | 1 |
|---|---|---|
| $\rightarrow q_0$ | $\{q_0, q_1\}$ | $\{q_0\}$ |
| $q_1$ | $\varnothing$ | $\{q_2\}$ |
| $*q_2$ | $\varnothing$ | $\varnothing$ |

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## Example

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## Definition

A NFA is quintuple $(Q, \Sigma, \Delta, I, F)$ such that

1. $Q$ is a set of states
2. $\Sigma$ is an alphabet
3. $\Delta$ is relation on $(Q \times \Sigma) \times Q$
4. $I$ are the initial states
5. $F$ are the final states

## Example

```
accept(S) :-
    initial(Q),
    accept(Q,S).
accept(Q,[X|Xs]) :-
    delta(Q,X,Q1),
    accept(Q1,Xs).
accept(Q,[]) :-
    final(Q).
```

## Example

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accept(Q,[X|Xs]) :-
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accept(Q,[]) :-
    final(Q).
initial(q0).
final(q2).
```

## Example

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accept(S) :-
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accept(Q,[]) :-
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initial(q0).
final(q2).

delta(q0,0,q0).
delta(q0,0,q1).
delta(q0,1,q0).
delta(q1,1,q2).
```

## Example

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initial(q0).
final(q2).

delta(q0,0,q0).
delta(q0,0,q1).
delta(q0,1,q0).
delta(q1,1,q2).

:- accept([0,0,0,1,0,1]).
```

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## Recall

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- `integer`: type check for an integer
- `atom`: type check for an atom
- `compound`: type check for a compound term

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## Definition

- `is_list`: type check for a list
- `integer`: type check for an integer
- `atom`: type check for an atom
- `compound`: type check for a compound term

## Example

```
constant(X) :-
    integer(X).
constant(X) :-
    atom(X).
```

## Example

```
flatten([X|Xs],Ys) :-
    is_list(X), flatten(X,Ys1),
    flatten(Xs,Ys2), append(Ys1,Ys2,Ys).
```

## Example

```
flatten([X|Xs],Ys) :-
    is_list(X), flatten(X,Ys1),
    flatten(Xs,Ys2), append(Ys1,Ys2,Ys). flatten(X,[X]) :- 
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:- flatten([[a],[b,[c,d]],e],[a,b,c,d,e])
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## Example

```
flatten(Xs,Ys) :- flatten(Xs,[],Ys).
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flatten([],[]).
:- flatten([[a],[b,[c,d]],e],[a,b,c,d,e])
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## Example

```
flatten(Xs,Ys) :- flatten(Xs,[],Ys).
flatten([X|Xs],S,Ys) :-
    is_list(X), flatten(X,[Xs|S],Ys).
```

## Example

```
flatten([X|Xs],Ys) :-
    is_list(X), flatten(X,Ys1),
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flatten([X|Xs],S,Ys) :-
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flatten([X|Xs],S,[X,Ys]) :-
    constant(X), X ≠ [], flatten(Xs,S,Ys).
```

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flatten([X|Xs],S,[X,Ys]) :-
    constant(X), X ≠ [], flatten(Xs,S,Ys).
flatten([],[X|S],Ys) :- flatten(X,S,Ys).
flatten([],[],[]).
```

# Once Again: Difference Lists and DCGs

## Example

```
:- listen_zusammen([[1,2],[],[4,5]],[1,2,4,5]).

listen_zusammen(Xss,Xs) :-
        phrase(seqq(Xss),Xs).
```

# Once Again: Difference Lists and DCGs

## Example

```
:- listen_zusammen([[1,2],[],[4,5]],[1,2,4,5]).

listen_zusammen(Xss,Xs) :-
        phrase(seqq(Xss),Xs).

seqq([]) -->
        [].
seqq([Xs|Xss]) -->
        seq(Xs),
        seqq(Xss).
```

# Once Again: Difference Lists and DCGs

## Example

```
:- listen_zusammen([[1,2],[],[4,5]],[1,2,4,5]).

listen_zusammen(Xss,Xs) :-
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seqq([]) -->
        [].
seqq([Xs|Xss]) -->
        seq(Xs),
        seqq(Xss).

seq([]) -->
        [].
seq([C|Cs]) -->
        [C],
        seq(Cs).
```

# GUPU

Example (definite clause grammars)

- Example 47 . . .