

Logic Programming

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Overview

Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, unification, database and recursive programming, termination

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, **constraint logic programming**

Full Prolog

semantics, correctness proofs, meta-logical predicates, cuts nondeterministic programming, efficient programs, complexity

Summary of Last Lecture

Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
      region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
      region(e,E,[B,C,F]), region(f,F,[C,D,E]))].
```

```
coloured_map([Region|Regions], Colours) :-
  coloured_region(Region,Colours),
  coloured_map(Regions,Colours).
coloured_map([],Colours).
```

```
coloured_region(region(Name,Colour,Neighbours), Colours) :-
  select(Colour,Colours,Colours1),
  sublist_of(Neighbours,Colours1).
```

```
test_colour(Map) :-
  is_map(Map),
  is_colours(Colours),
  coloured_map(Map,Colours).
```

Cryptarithmic

Definition

- a cryptarithmic problem is a puzzle in which each letter represents a unique digit ≤ 9
- the object is to find the value of each letter
- first digit cannot be 0

Example

$$\begin{array}{r} \text{S E N D} \\ \text{M O R E} \\ \hline \text{M O N E Y} \end{array} +$$

First Attempt

generate and test

```
solve ([[S,E,N,D],[M,O,R,E],[M,O,N,E,Y]]) :-
    Digits = [D, E, M, N, O, R, S, Y],
    Carries = [C1,C2,C3,C4],
    selects(Digits, [0,1,2,3,4,5,6,7,8,9]),
    members(Carries, [0,1]),
    M      == C4,
    O + 10 * C4 == S + M + C3,
    N + 10 * C3 == E + O + C2,
    E + 10 * C2 == N + R + C1,
    Y + 10 * C1 == D + E,
    M > 0, S > 0.

:- solve(X),
X = [[9, 5, 6, 7], [1, 0, 8, 5], [1, 0, 6, 5, 2]].
```

Discussion

very inefficient

```
?- time(solve(X)).
% 133,247,057 inferences ,
% 7.635 CPU in 7.667 seconds (100% CPU, 17452690 Lips)
X = [[9, 5, 6, 7], [1, 0, 8, 5], [1, 0, 6, 5, 2]]
```

explanation

- generate-and-test in it's purest form
- all guesses are performed before the constraints are checked
- arithmetic checks cannot deal with variables

improvement

- move testing into generating
- destroys clean structure of program
- any other ideas?

Constraint Logic Programming

Definitions (CLP on finite domains)

- `use_module(library(clpfd))` loads the clpfd library
- `Xs ins N .. M` specifies that all values in `Xs` must be in the given range
- `all_different(Xs)` specifies that all values in `Xs` are different
- `label(Xs)` all variables in `Xs` are evaluated to become values
- `#=`, `#\=`, `#>`, ... like the arithmetic comparison operators, but may contain (constraint) variables

standard approach

- load the library
- specify all constraints
- call `label` to start efficient computation of solutions

Second Attempt

constraint logic program

```
solve ([[S,E,N,D],[M,O,R,E],[M,O,N,E,Y]]) :-
    Digits = [D, E, M, N, O, R, S, Y],
    Carries = [C1,C2,C3,C4],
    Digits ins 0 .. 9, all_different(Digits),
    Carries ins 0 .. 1,
    M      #= C4,
    O + 10 * C4 #= S + M + C3,
    N + 10 * C3 #= E + O + C2,
    E + 10 * C2 #= N + R + C1,
    Y + 10 * C1 #= D + E,
    M #> 0, S #> 0,
    label(Digits).
```

8 queens (as before)

```
queens(Xs) :- template(Xs), solution(Xs).

template([1/_Y1,2/_Y2,3/_Y3,4/_Y4,
          5/_Y5,6/_Y6,7/_Y7,8/_Y8]).

solution([]).
solution([X/Y|Others]) :-
    solution(Others),
    member(Y, [1,2,3,4,5,6,7,8]),
    noattack(X/Y, Others).

noattack(_, []).
noattack(X/Y, [X1/Y1|Others]) :-
    Y \= Y1,
    Y1 - Y \= X1 - X,
    Y1 - Y \= X - X1,
    noattack(X/Y, Others).
```

 n -queens (using clp)

```
nqueens(N, Qs) :-
    length(Qs, N),
    Qs ins 1 .. N, all_different(Qs),
    constraint_queens(Qs),
    label(Qs).

constraint_queens([]).
constraint_queens([Q|Qs]) :-
    noattack(Q, Qs, 1),
    constraint_queens(Qs).

noattack(_, [], _).
noattack(X, [Q|Qs], N) :-
    X #\= Q+N,
    X #\= Q-N,
    M is N+1,
    noattack(X, Qs, M).
```

Definition

- **Sudoku** is a well-known logic puzzle; usually played on a 9×9 grid
- \forall *cells*: $cells \in \{1, \dots, 9\}$
- \forall *rows*: all entries are different
- \forall *columns*: all entries are different
- \forall *blocks*: all entries are different

Main Loop (using clp)

```
sudoku(Puzzle) :-
    show(Puzzle),
    flatten(Puzzle, Cells),
    Cells ins 1 .. 9,
    rows(Puzzle),
    cols(Puzzle),
    blocks(Puzzle),
    label(Cells),
    show(Puzzle).
```

auxiliary predicates

- *flatten*/2 flattens a list
- *show*/1 prints the current puzzle

row/1

```
rows([]).
rows([R|Rs]) :-
    all_different(R), rows(Rs).
```

row/1 (alternative)

```
rows(Rs) :- maplist(all_distinct, Rs).
```

```

cols/1
cols ([[ | -]).
cols ([
    [X1|R1],
    [X2|R2],
    [X3|R3],
    [X4|R4],
    [X5|R5],
    [X6|R6],
    [X7|R7],
    [X8|R8],
    [X9|R9]]) :-
    all_different([X1,X2,X3,X4,X5,X6,X7,X8,X9]),
    cols([R1,R2,R3,R4,R5,R6,R7,R8,R9]).

```

cols/1 (alternative)

use *maplist/2*

```

blocks/1
blocks ([]).
blocks ([[ ], [ ], [ ] | Rs]) :- blocks(Rs).
blocks ([[X1,X2,X3|R1],
        [X4,X5,X6|R2],
        [X7,X8,X9|R3] | Rs]) :-
    all_different([X1,X2,X3,X4,X5,X6,X7,X8,X9]),
    blocks([R1,R2,R3 | Rs]).

```

Example

```

:- sudoku([[1,--,--,--,--,--,--],
          [--,2,7,4,--,--,--],
          [--,--,5,--,--,4],
          [--,3,--,--,--,--],
          [7,5,--,--,--,--],
          [--,--,9,6,--,--],
          [--,4,--,6,--,--],
          [--,--,--,7,1,--],
          [--,--,1,3,--,--]).

```

GUPU

Example (constraint logic programming)

- Example 53, 58 ...