

# Functional Programming

## Exercises Week 11

(for January 13, 2017)

1. Read Chapter 9 of the lecture notes.
2. a) Supply the omitted parentheses in the following  $\lambda$ -terms.
  - $x_0 x_1 x_2 x_3$
  - $\lambda y. x_0 x_1 x_2 x_3$
  - $\lambda y. x_0 x_1 (\lambda x_2. x_2) x_3$b) Reduce the following  $\lambda$ -terms to  $\beta$ -normal form.
  - $(\lambda z_1 z_2 z_3. z_1 z_2 z_3) x_1 x_2 x_3$
  - $\lambda y. (\lambda z_1 z_2 z_3. z_1 z_2 z_3) x_1 x_2 x_3$
  - $\lambda y. (\lambda z_1 z_2 z_3. z_1 z_2 z_3) x_1 (\lambda x_2. x_2) x_3$
3. Exercise 9.3
4. Solve the unification problem given by the following equations

$$\alpha_1 \approx \alpha_2 \rightarrow \alpha_3 \rightarrow \alpha_4 \tag{a}$$

$$\alpha_2 \approx \text{list}(\alpha_5) \tag{b}$$

$$\alpha_4 \approx \alpha_3 \tag{c}$$

$$\alpha_3 \approx \text{list}(\alpha_6) \tag{d}$$

$$\alpha_7 \approx \alpha_8 \rightarrow \text{list}(\alpha_8) \rightarrow \text{list}(\alpha_8) \tag{e}$$

$$\alpha_9 \approx \alpha_0 \rightarrow \text{list}(\alpha_0) \rightarrow \text{list}(\alpha_0) \tag{f}$$

$$\alpha_0 \approx \alpha_8 \tag{g}$$

$$\alpha_2 \approx \text{list}(\alpha_8) \tag{h}$$

$$\text{list}(\alpha_8) \approx \alpha_4 \tag{i}$$

$$\alpha_4 \approx \text{list}(\alpha_0) \tag{k}$$

5. ( $\star$ ) Define the type checking and type inference rules for a let-rec expression:

`let rec f x = expr in f`

and to a let expression with multiple bindings:

`let x = expr and y = expr in expr`