Functional Programming in Isabelle/HOL

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Course: Functional Programming 703024, WS 2016/17

Overview

- Summary of Last Week
- Some Background
- Example: Knight Tours

Summary of Last Week

Theory File: `Summary.thy`

- + natural deduction
- mathematical induction
- recursive functions
- structural induction

Some Background

Theory File: `Insertion_Sort.thy`

- short history of Isabelle/HOL
- example – insertion sort

1969 **Dana Scott:**
Logic for Computable Functions

1969 1993

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Stanford LCF

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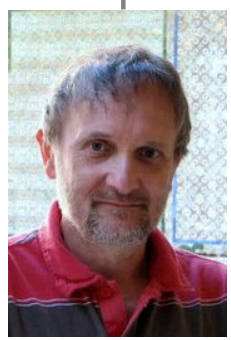
1986 **Isabelle**

1988 **Mike Gordon:**
HOL



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Tobias Nipkow:
Isabelle/HOL 1993

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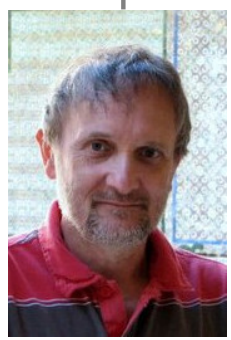

System Architecture

Standard ML implementation language

System Architecture

Isabelle/Pure generic proof assistant

Standard ML implementation language

System Architecture

Isabelle/HOL Higher-Order Logic

Isabelle/Pure generic proof assistant

Standard ML implementation language

System Architecture

Isabelle/jEdit jEdit based interface

Isabelle/Scala connects ML to JVM

Isabelle/HOL Higher-Order Logic

Isabelle/Pure generic proof assistant

Standard ML implementation language

A Small Trusted Kernel (aka the LCF approach)

- abstract type `thm` for theorems
- small set of trusted functions that create theorems
- no other way to create theorems

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```
sig
  type thm
  val assume : form -> thm
  val implies_intr : form -> thm -> thm
  val implies_elim : thm -> thm -> thm
                     ⋮
end
```

$$\frac{}{\phi \vdash \phi}$$

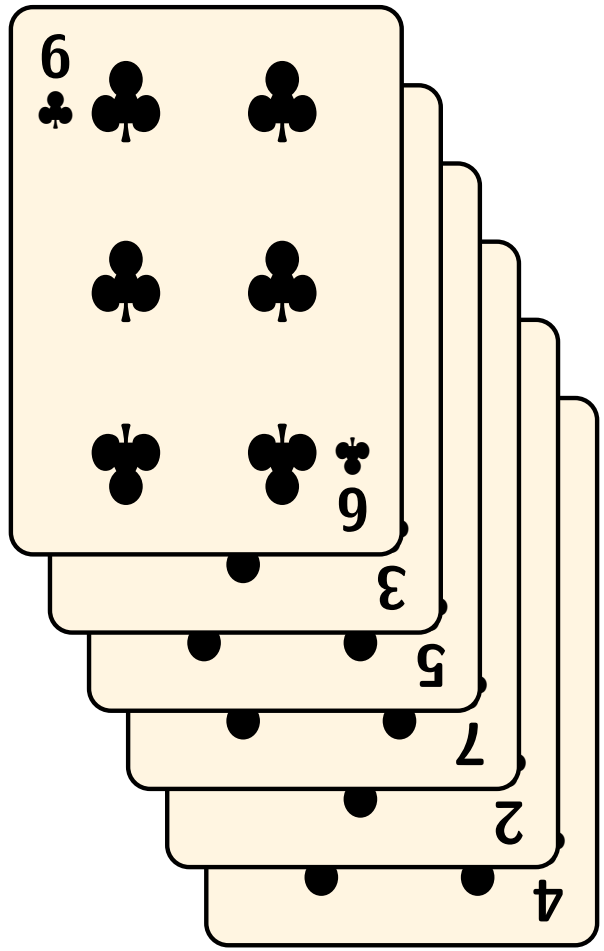
$$\frac{\Gamma \vdash \psi}{\Gamma \setminus \{\phi\} \vdash \phi \to \psi}$$

$$\frac{\Gamma \vdash \phi \quad \Gamma \vdash \phi \to \psi}{\Gamma \vdash \psi}$$

Higher-Order Logic

- HOL = Functional Programming + Logic
- data types (**`datatype`**)
- recursive functions (**`fun`**)
- logical connectives ($\wedge$, $\vee$, $\longrightarrow$, $\forall$, $\exists$, ...)

Example: Insertion Sort



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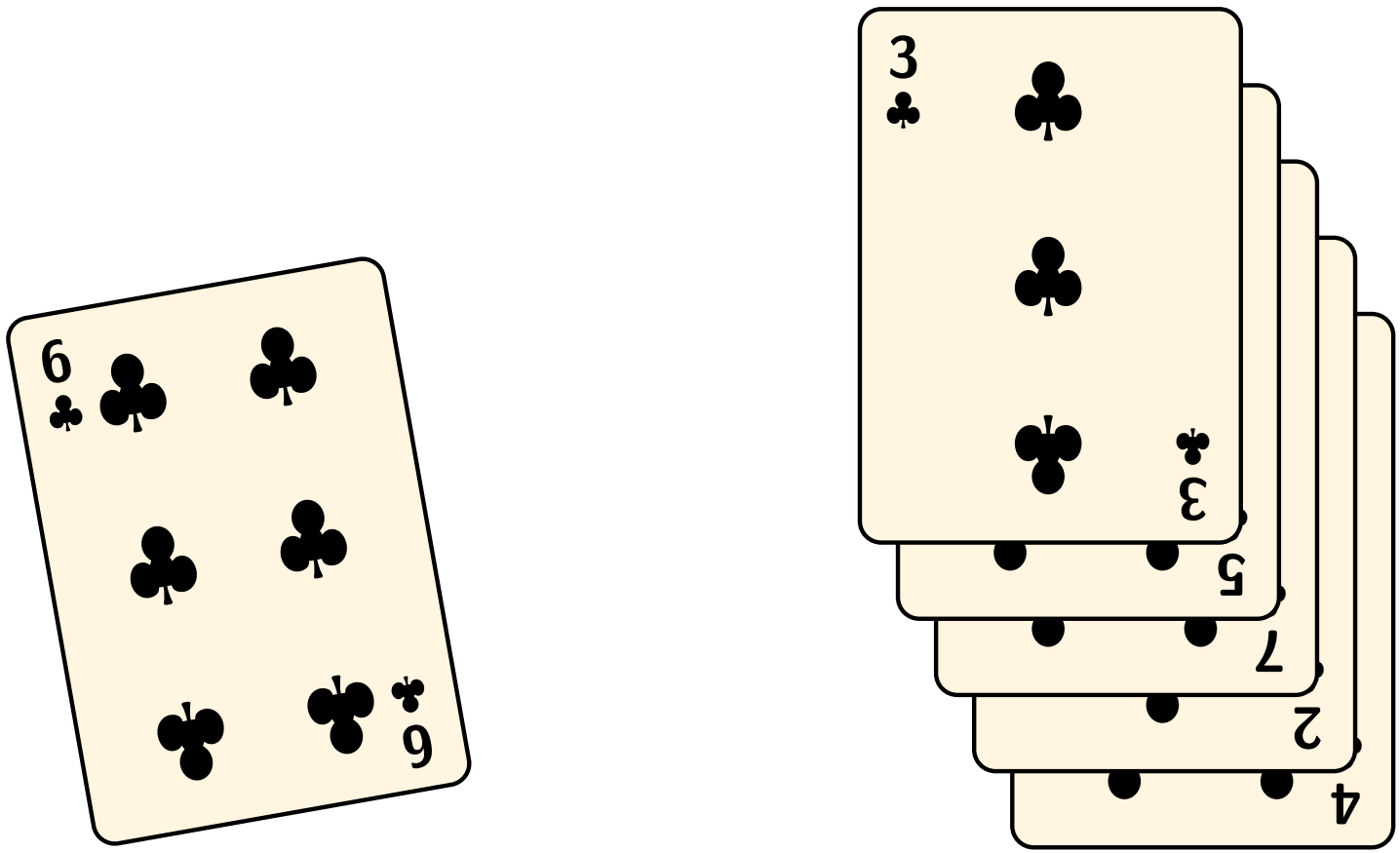

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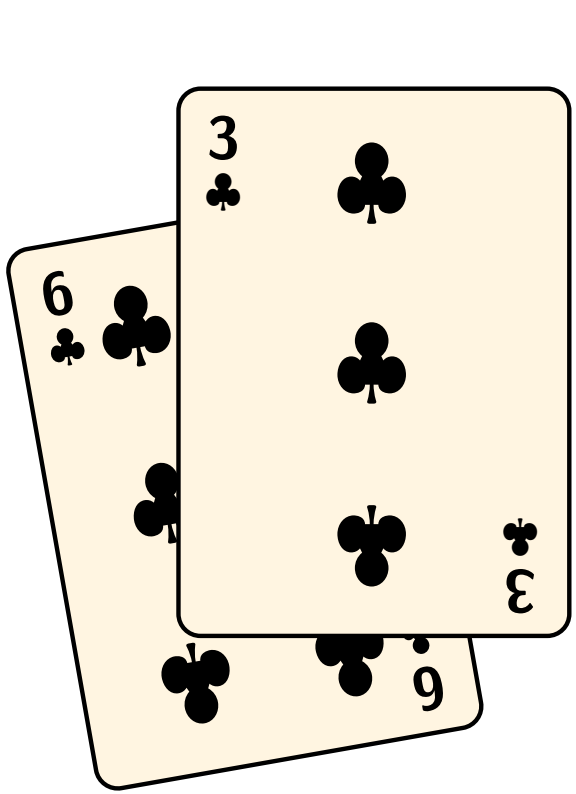

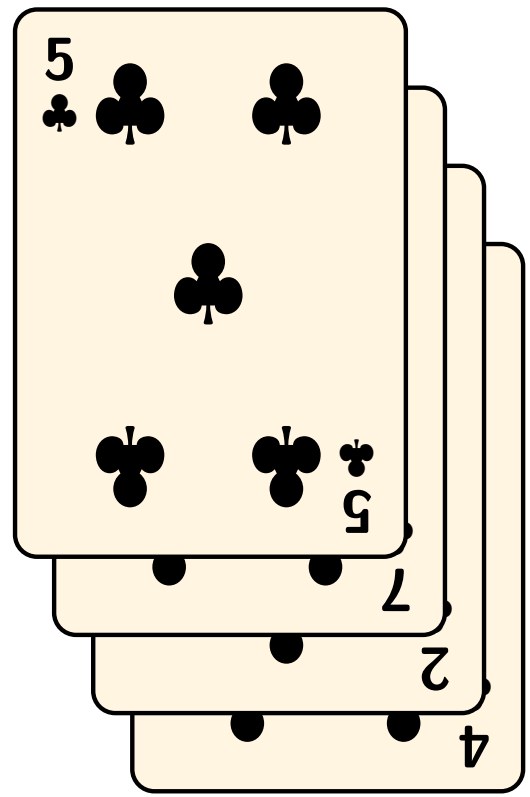

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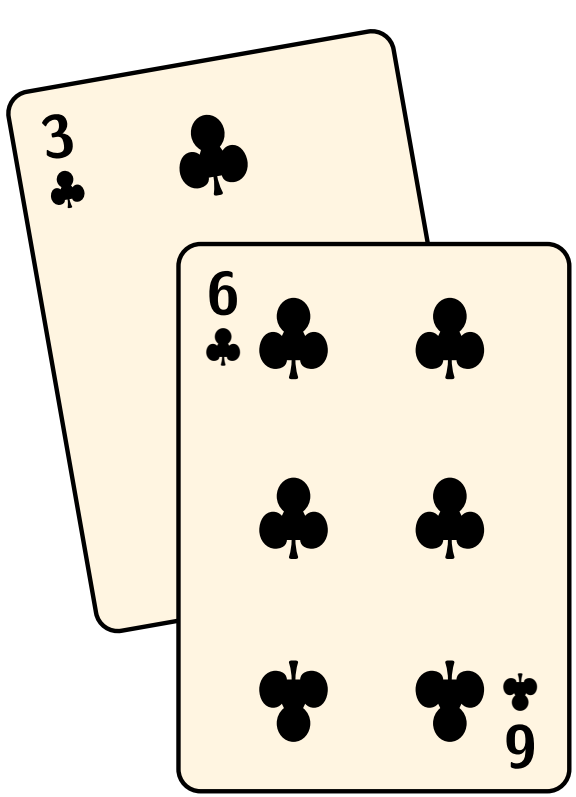

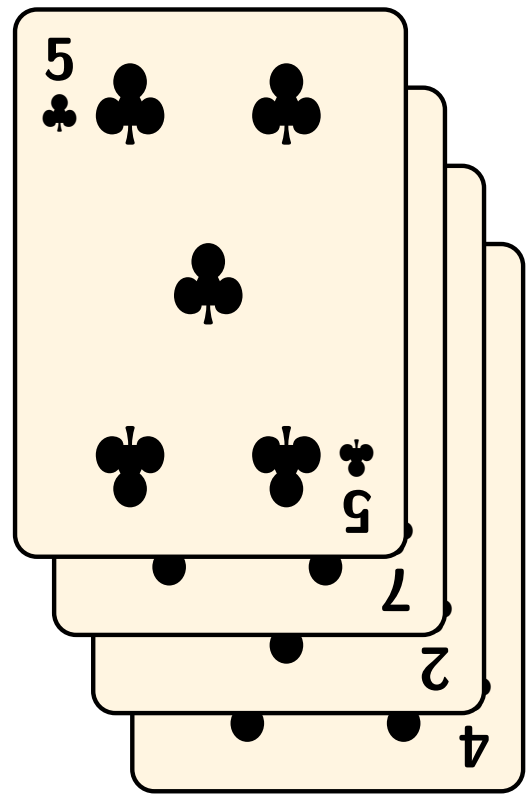

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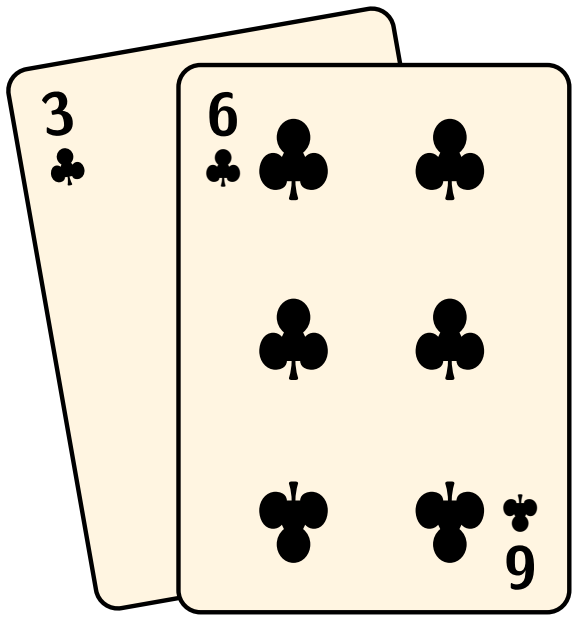

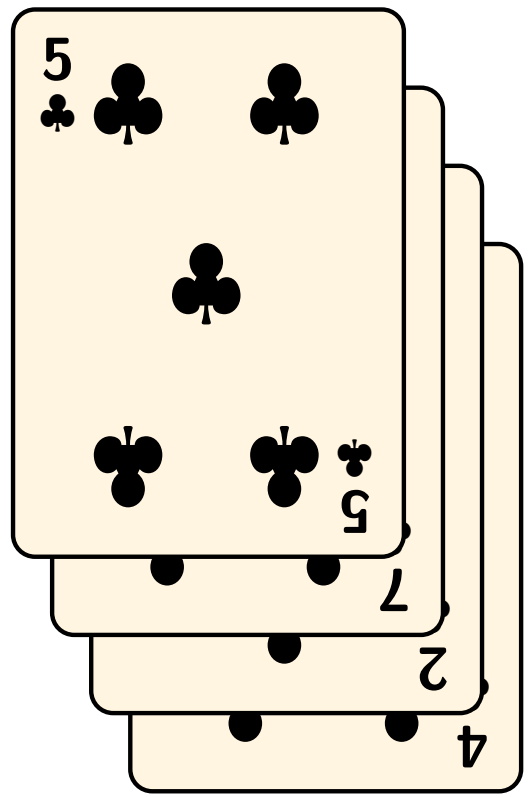

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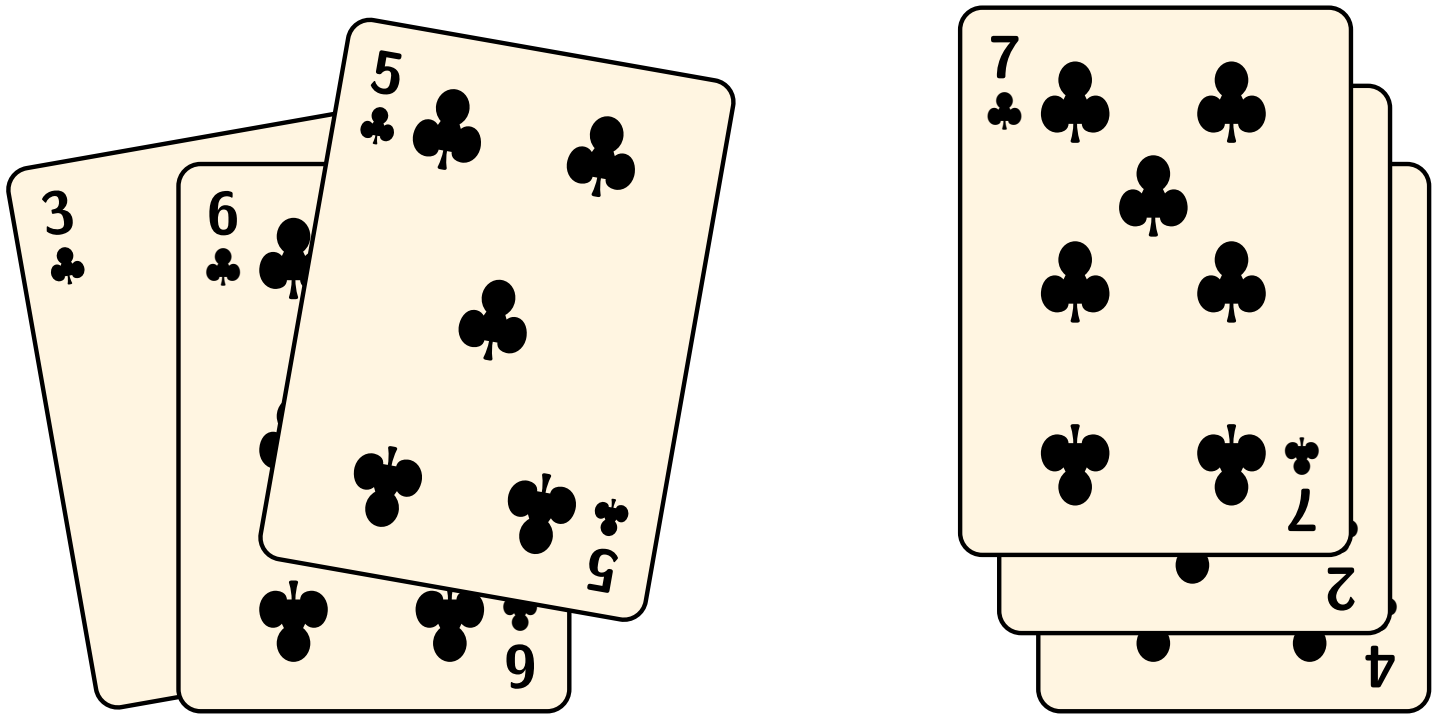

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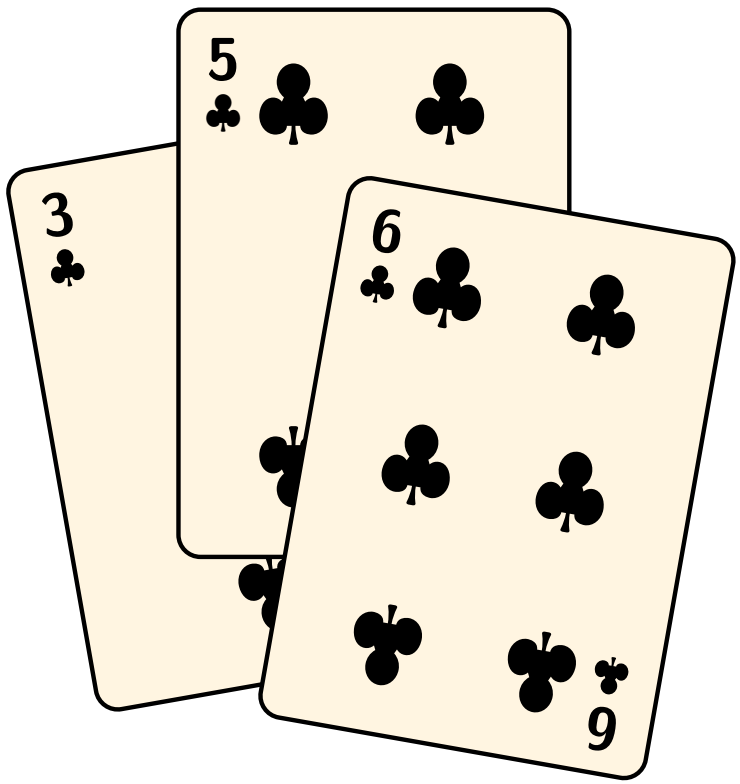

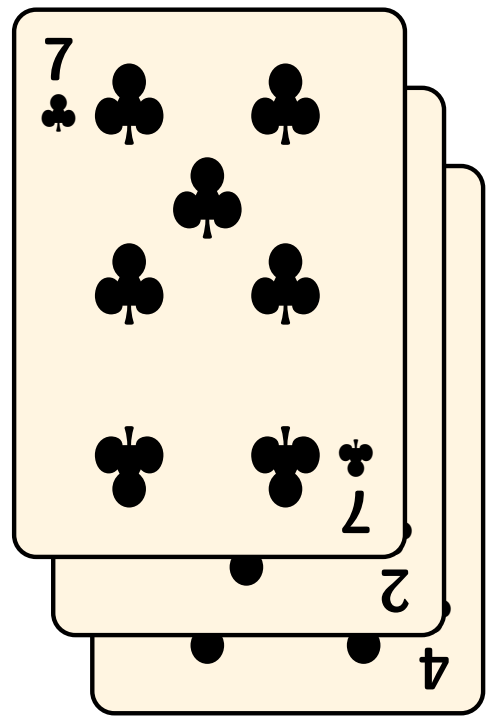

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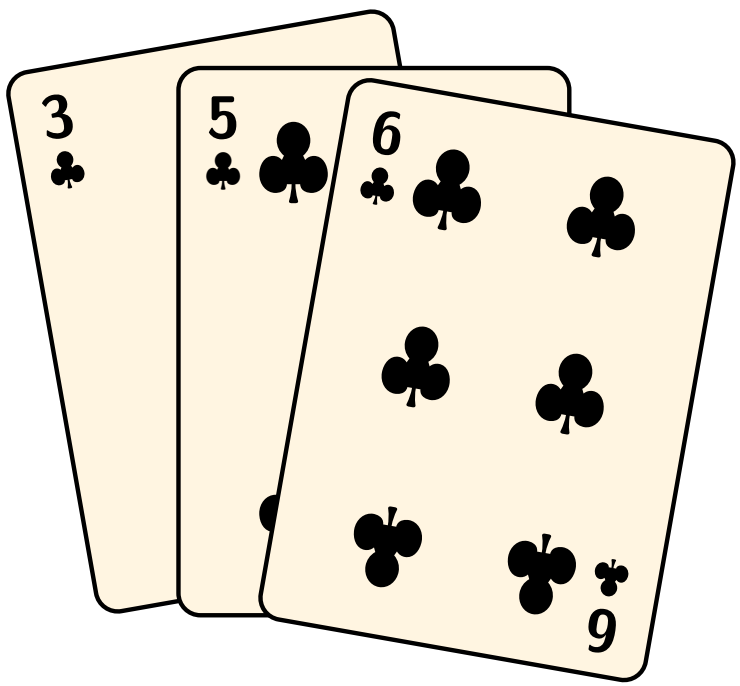

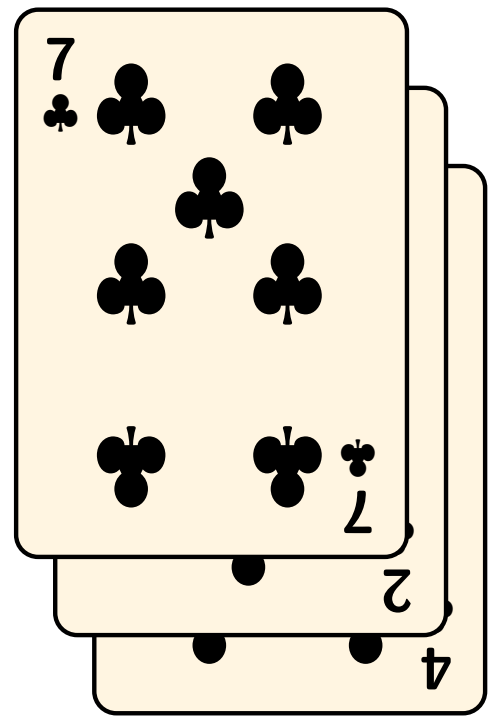

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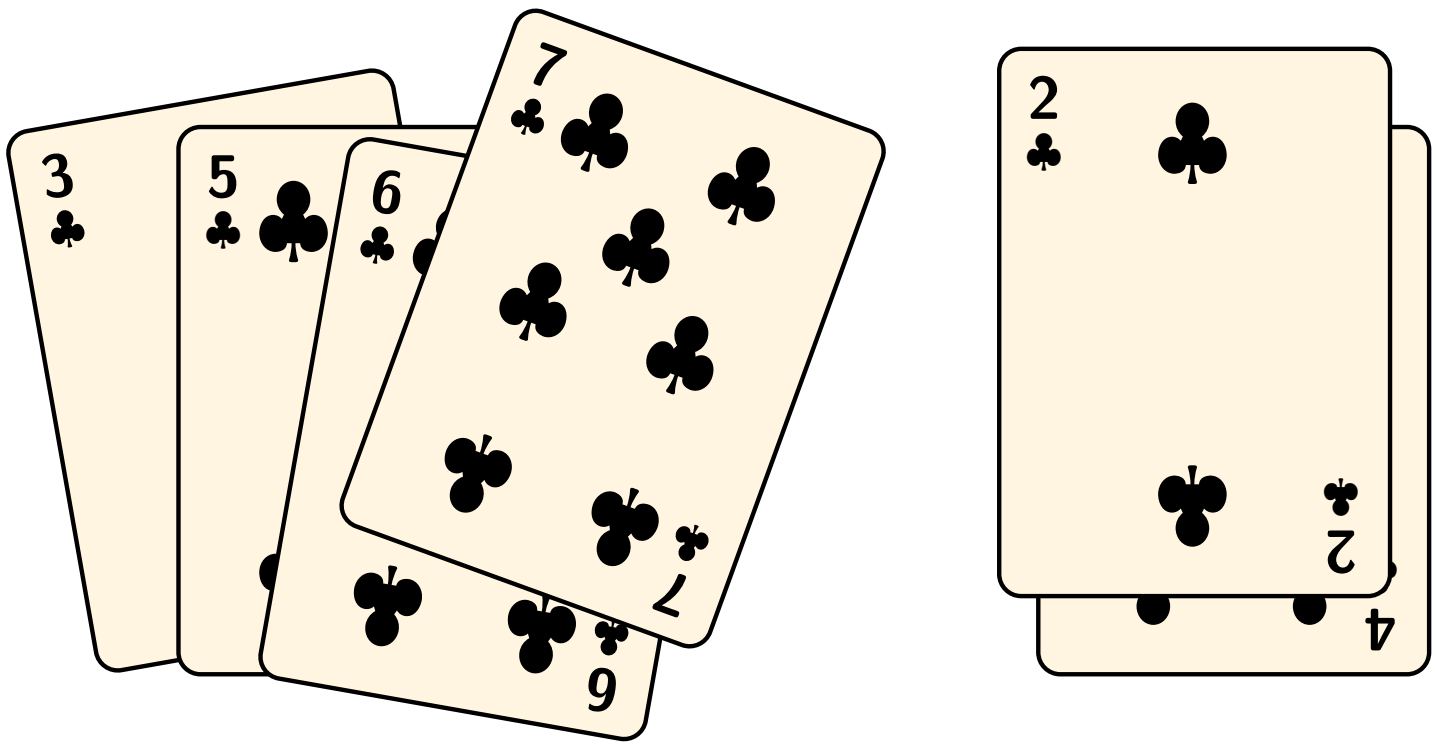

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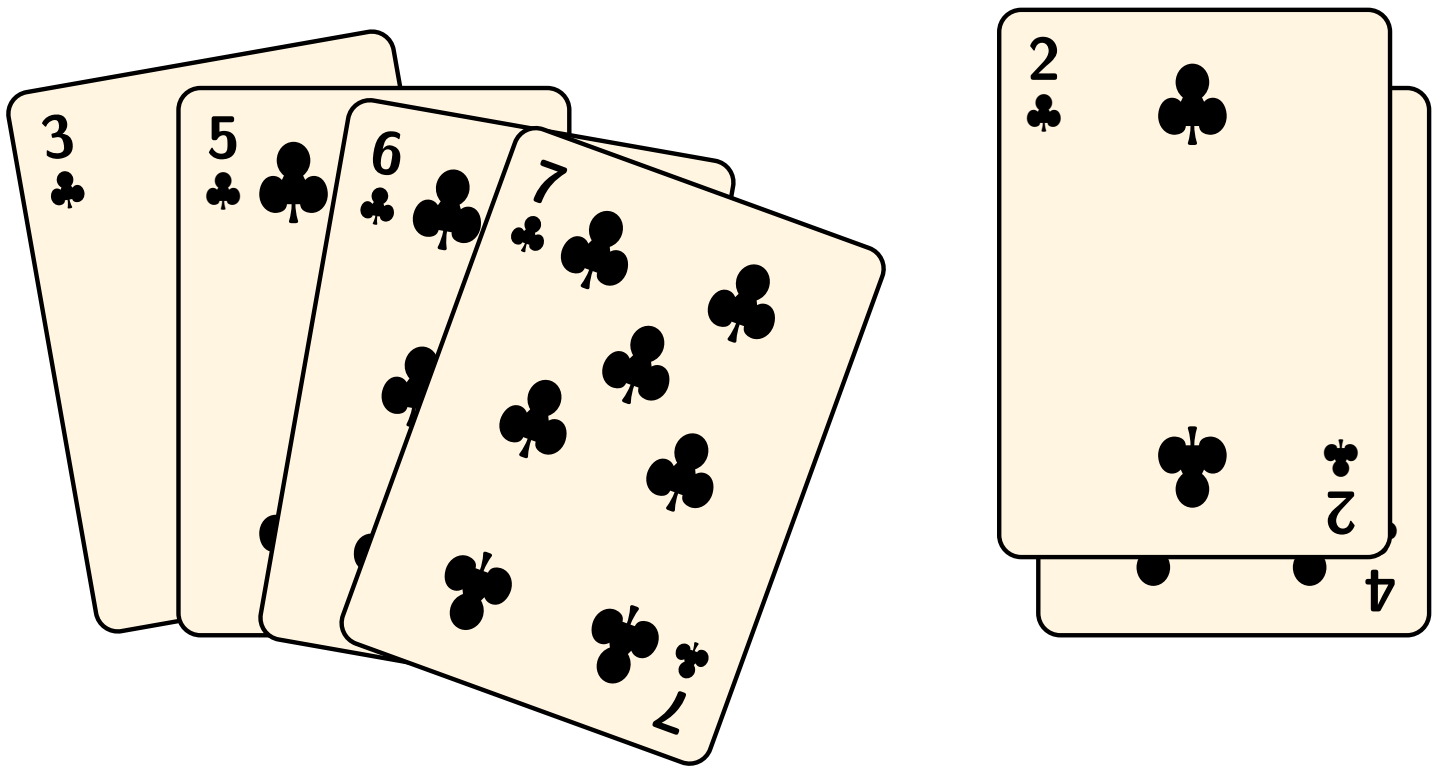

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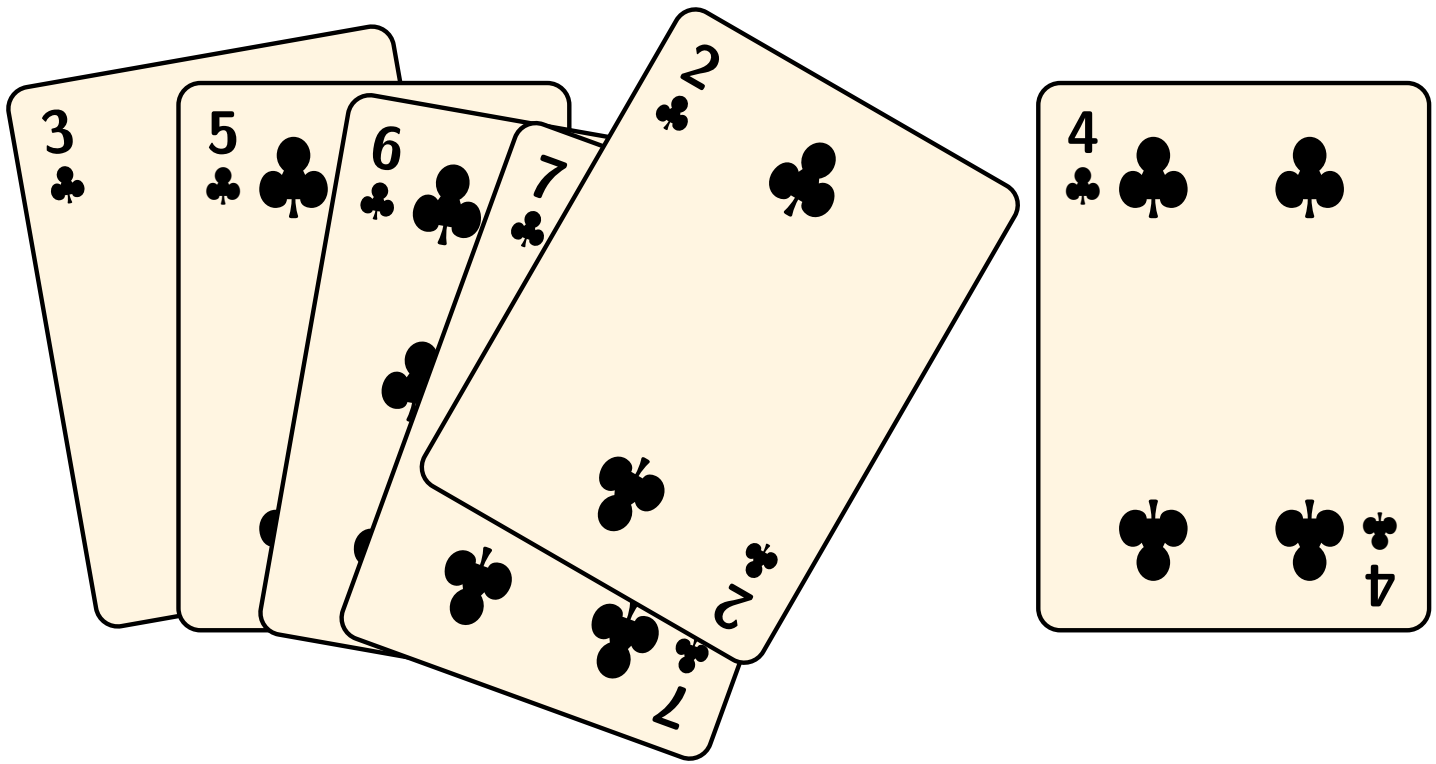

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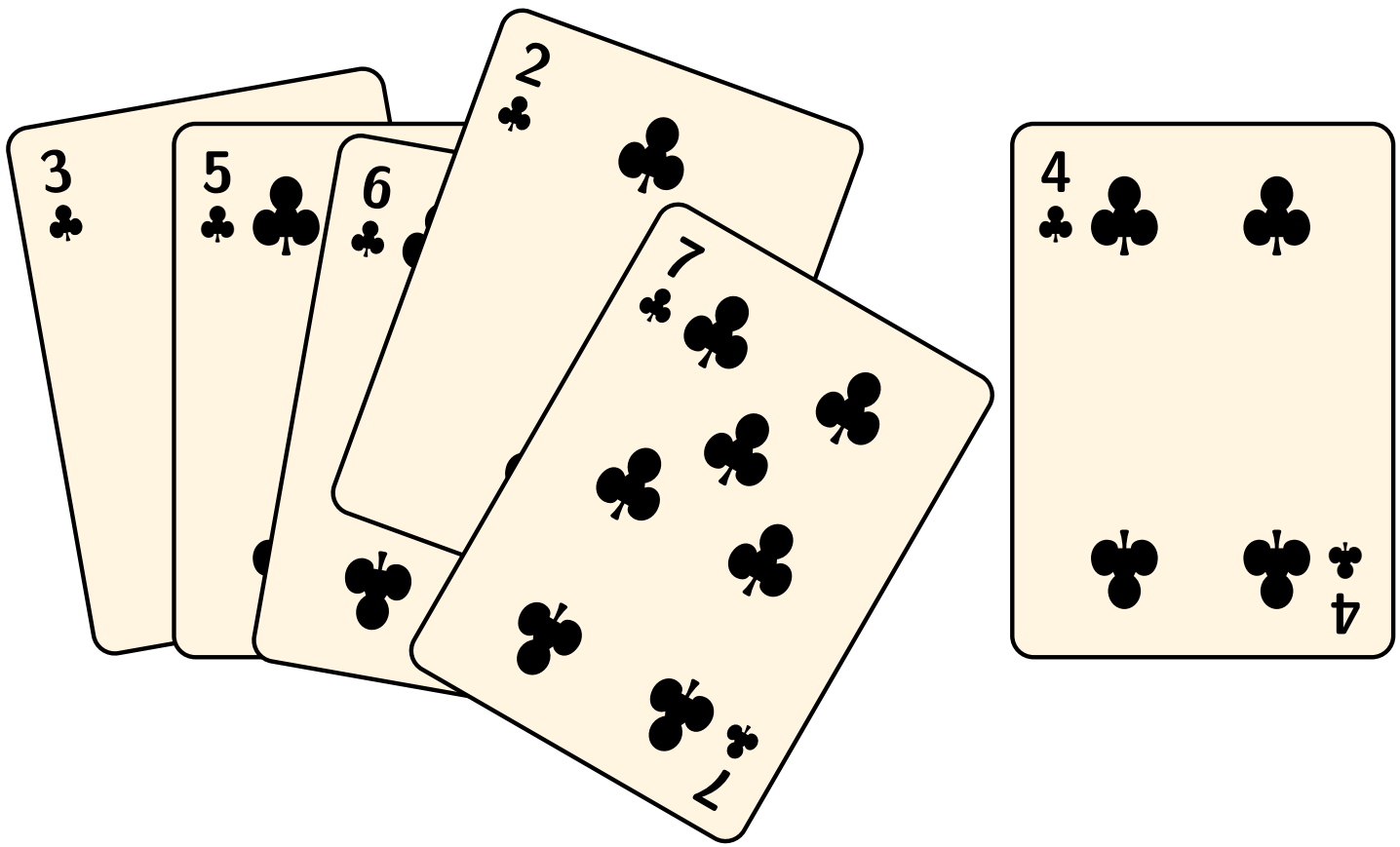

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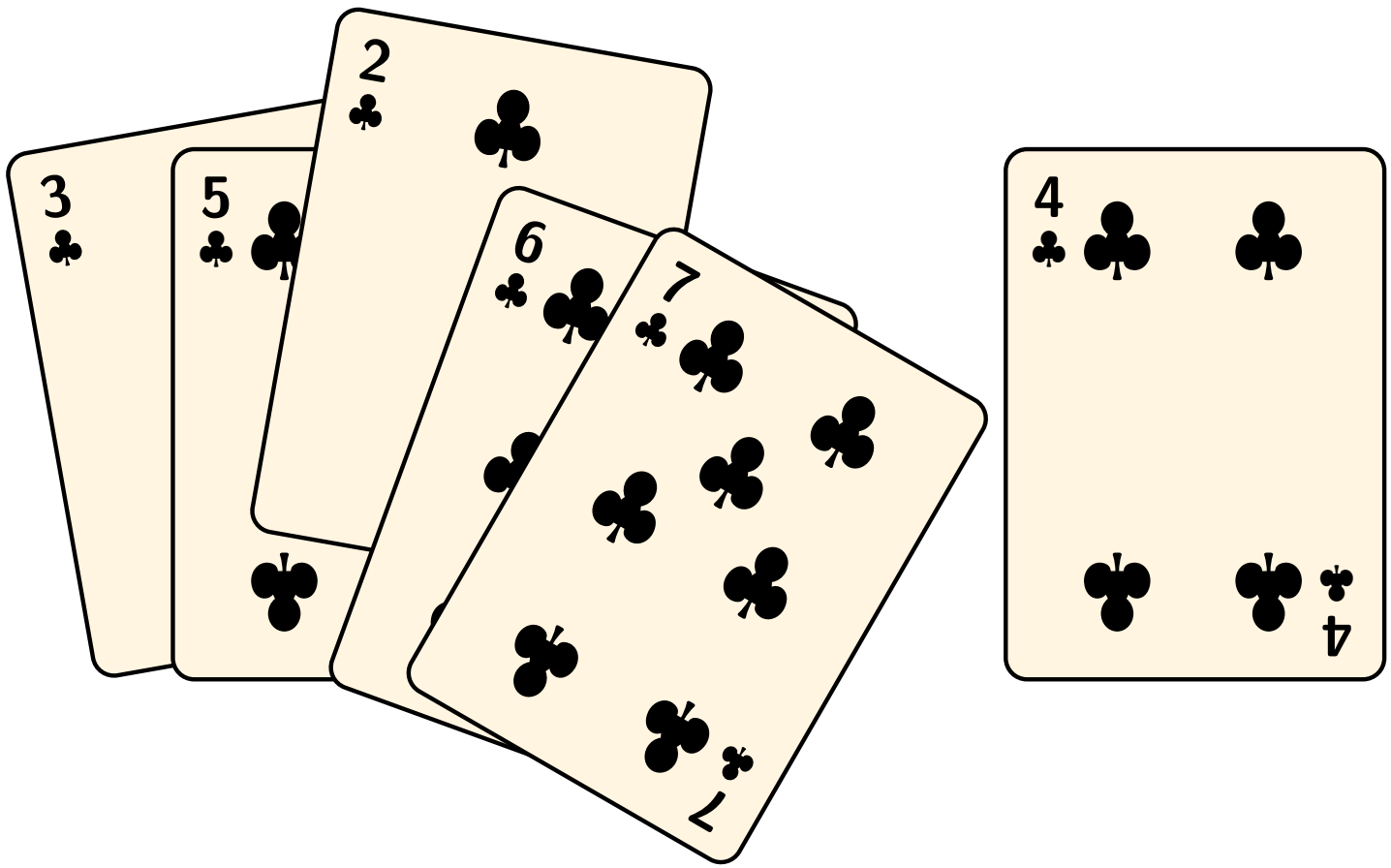

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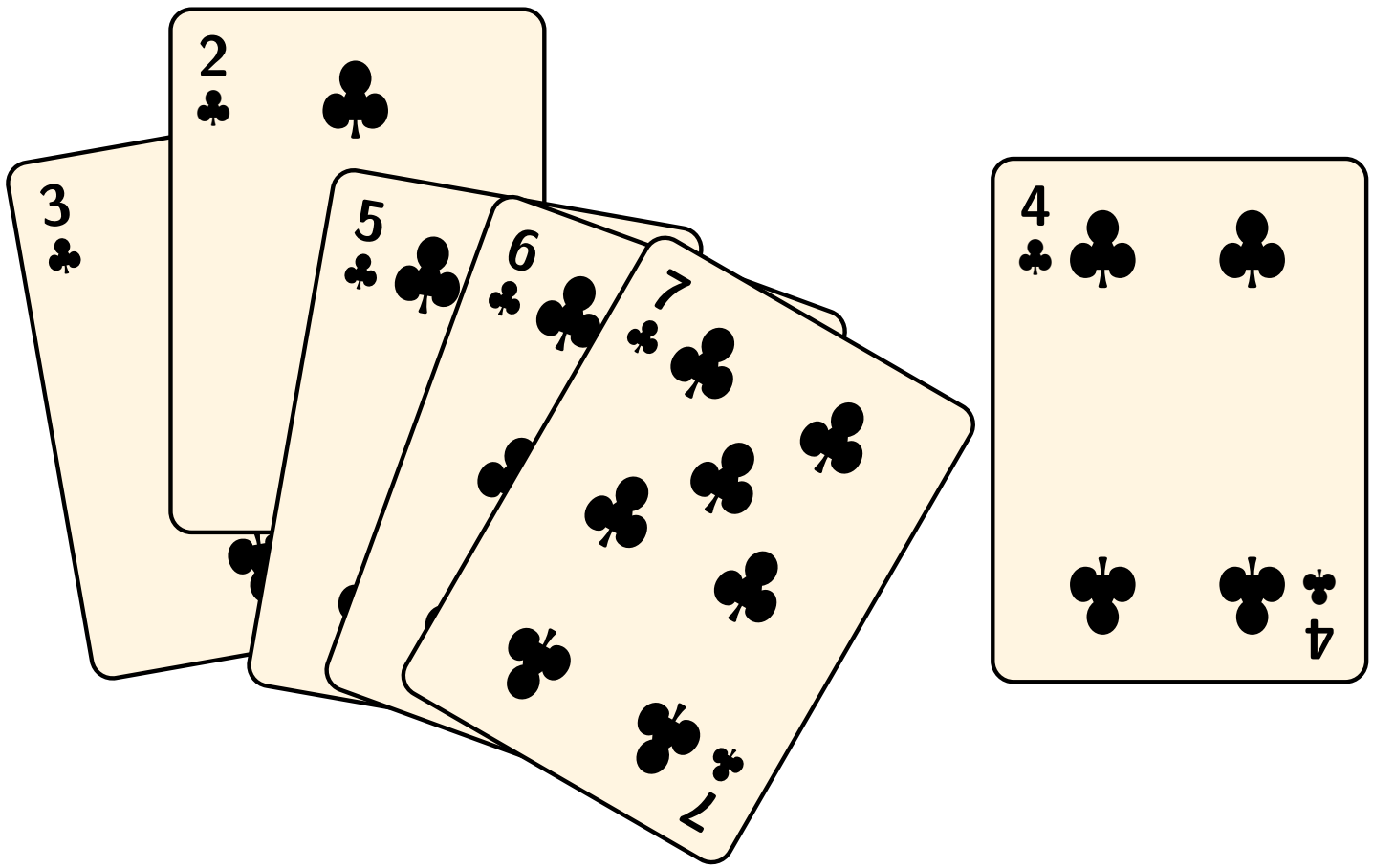

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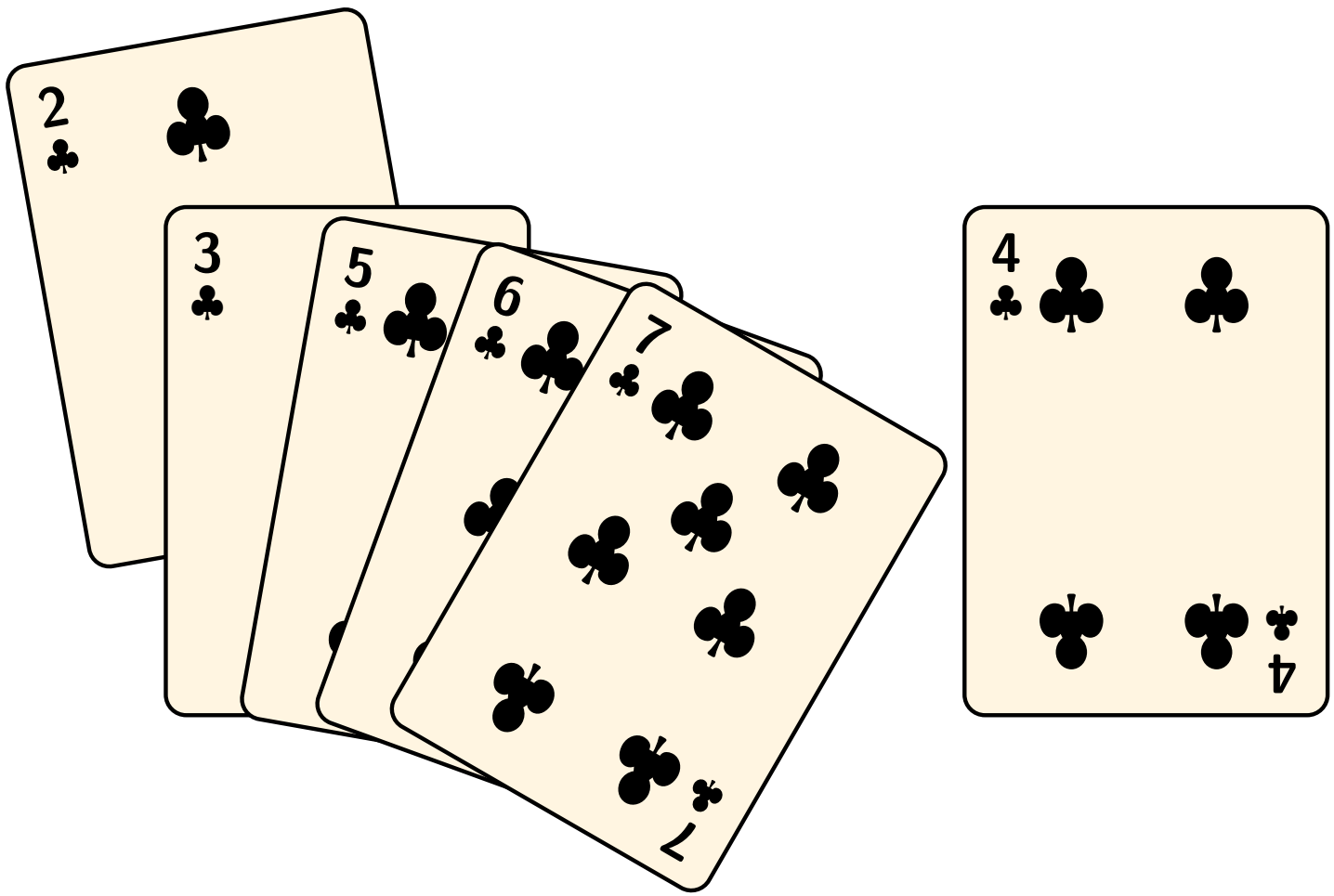

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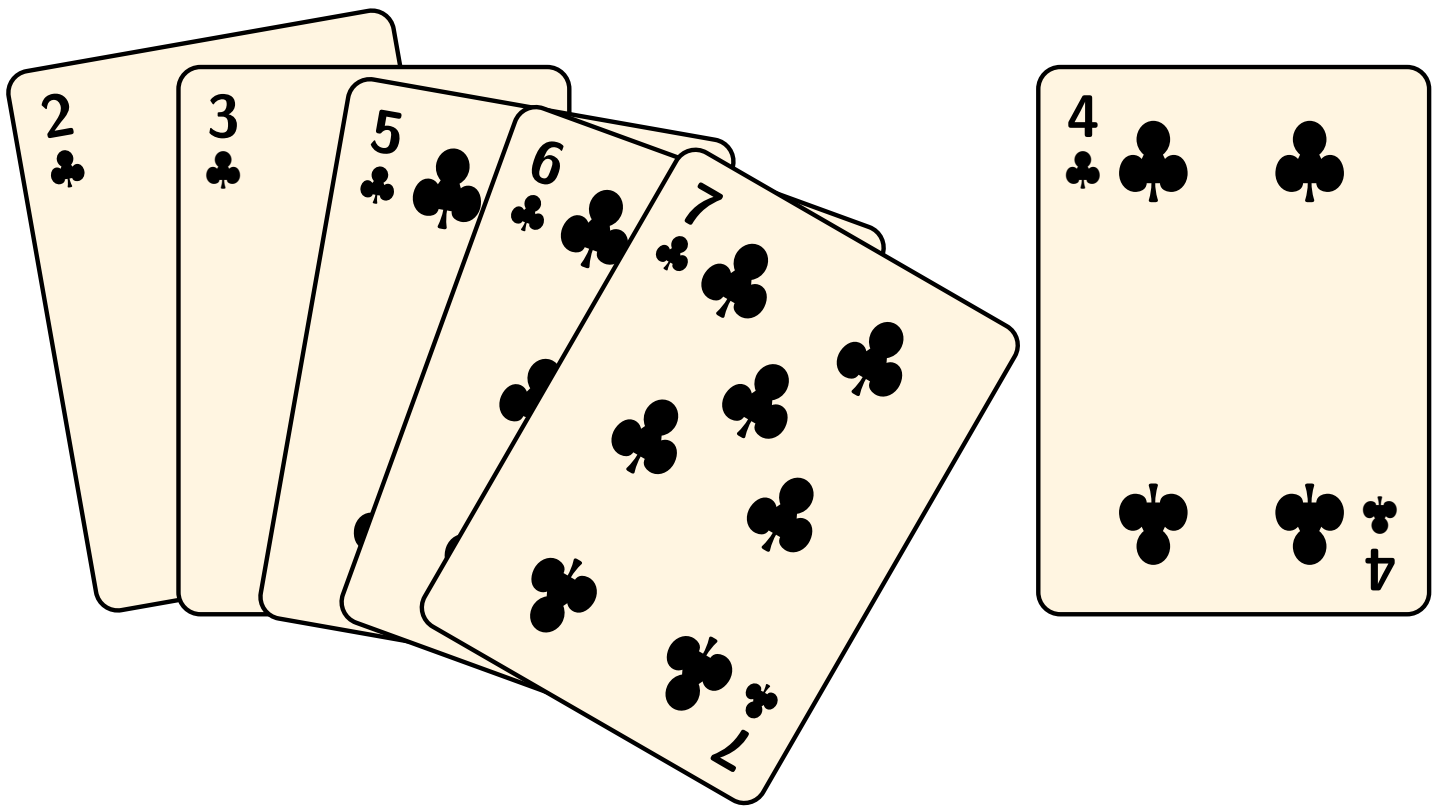

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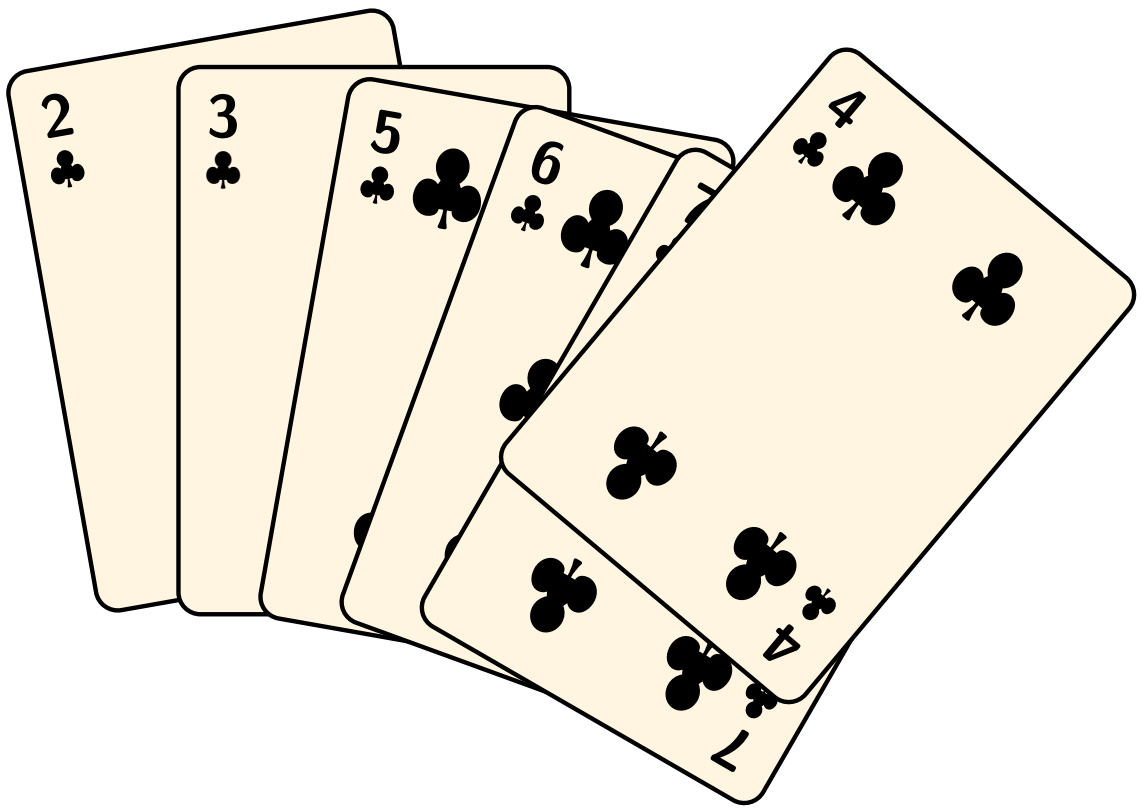

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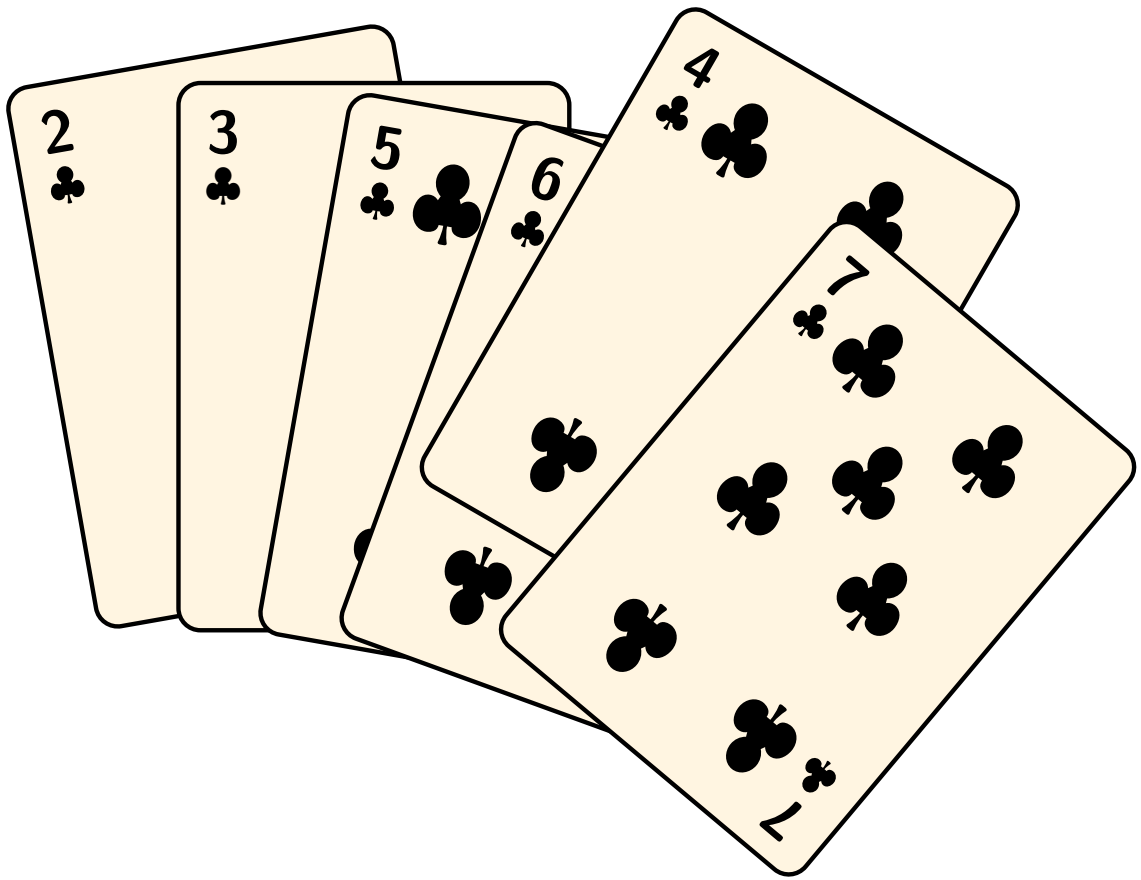

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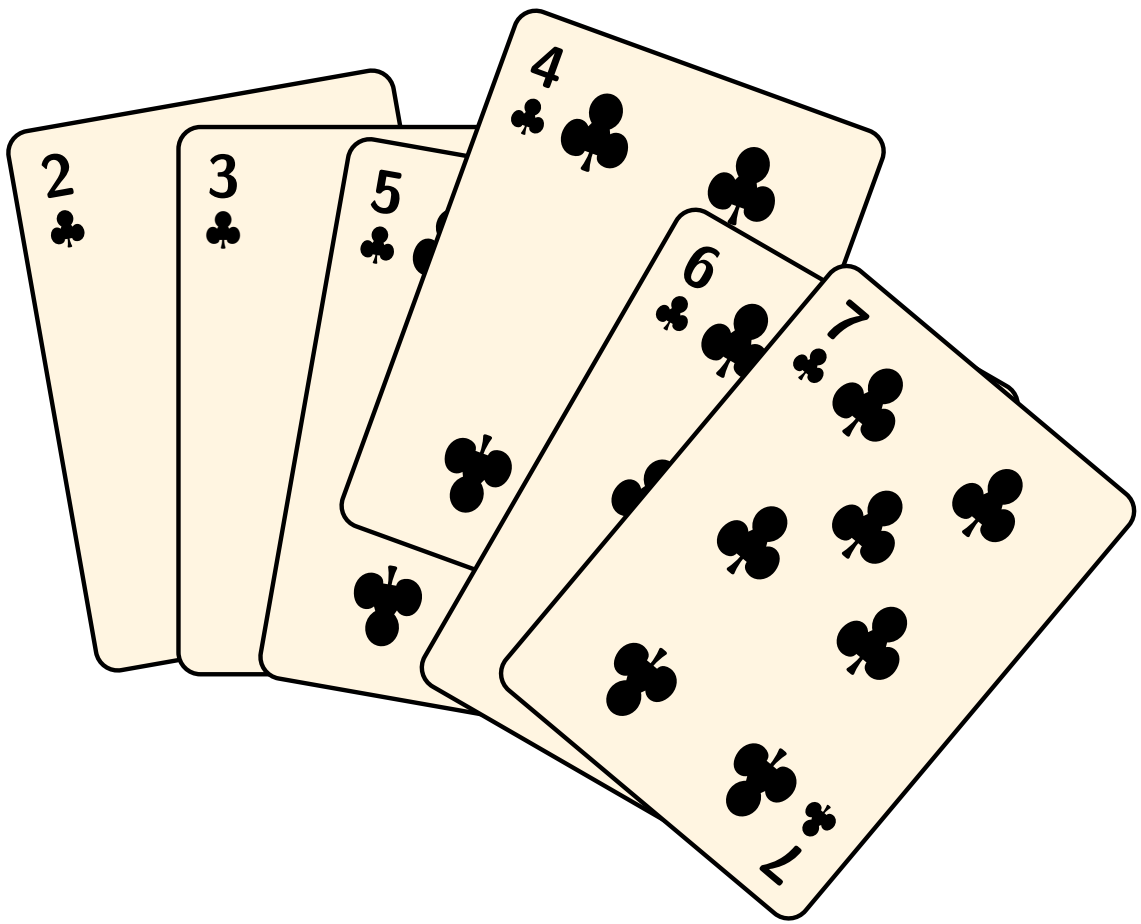

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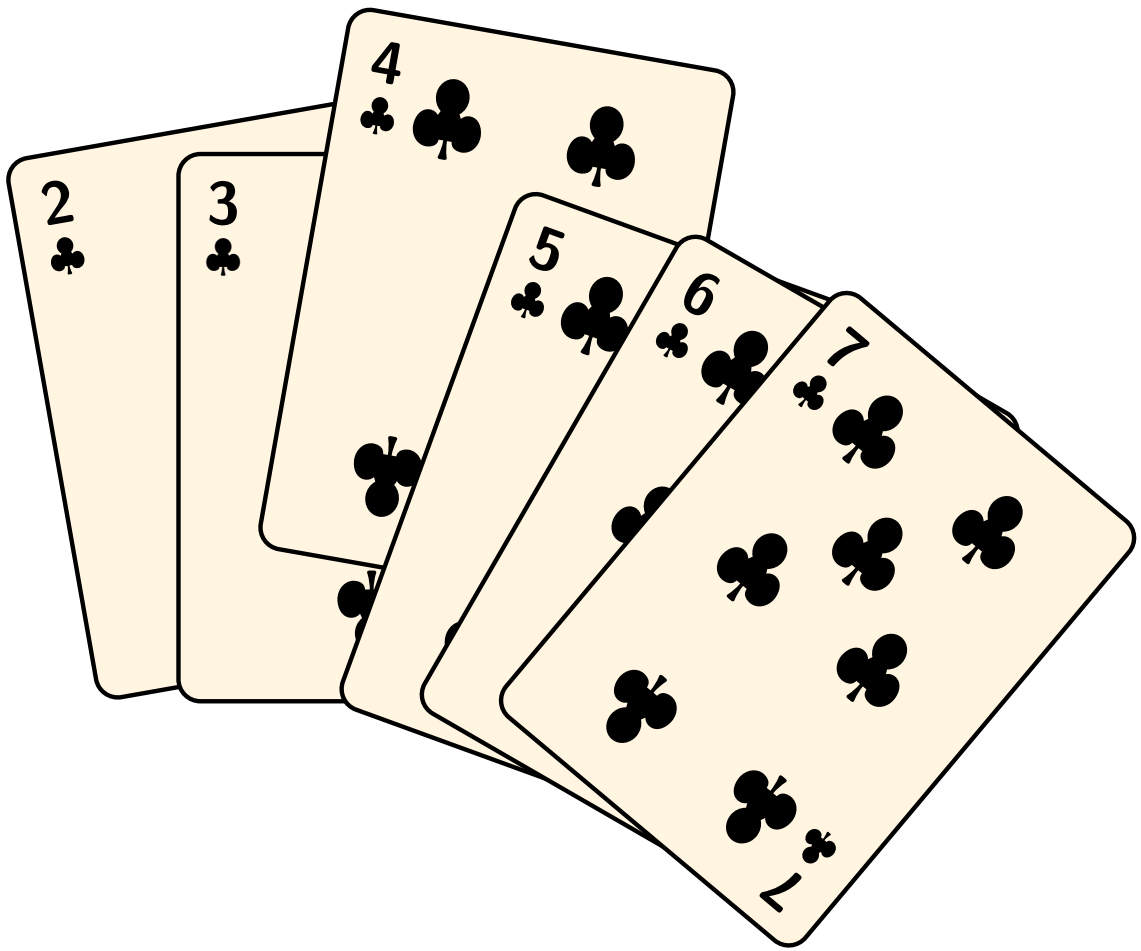

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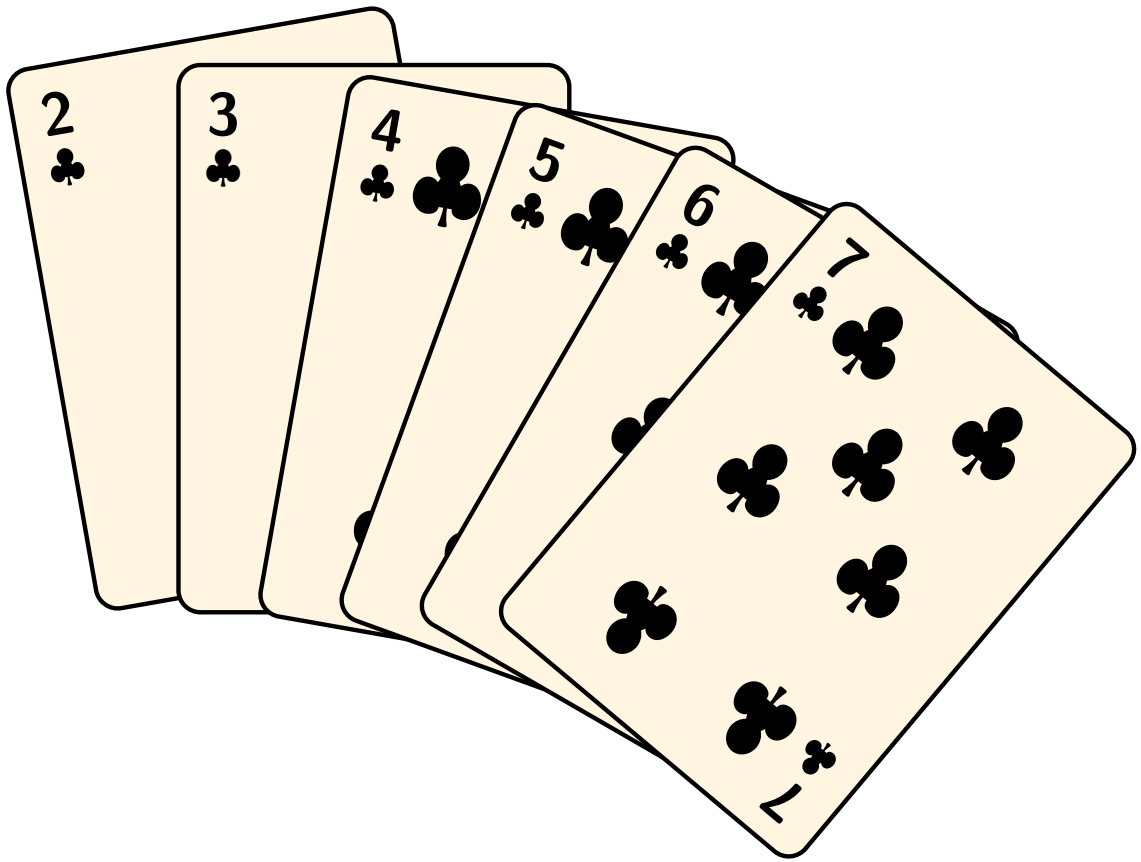

A Functional Implementation

- inserting an element into a sorted list

```
let rec insert x = function
| []     ->  [x]
| y::ys  ->  if x <= y then x::y::ys
             else y :: insert x ys
```

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```

- sorting by repeatedly inserting elements into the empty list

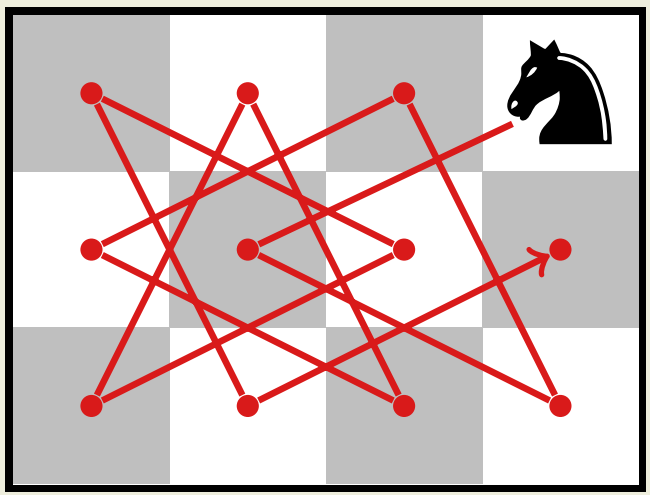
```
insertion_sort xs = List.fold_right insert xs []
```

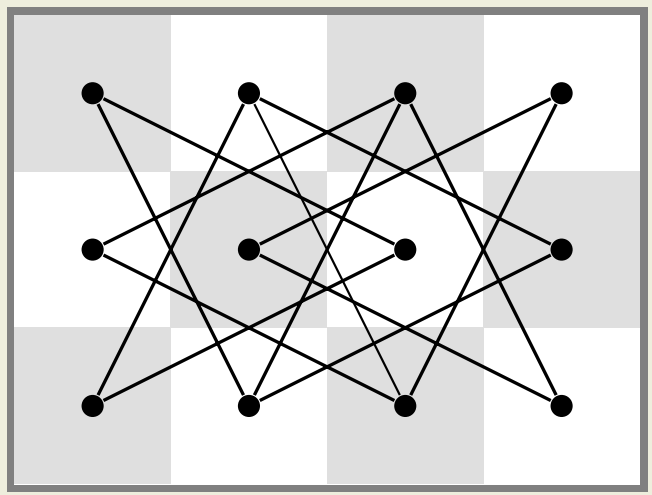
where

$$\texttt{List.fold\_right}\ f\ [x_1, x_2, \ldots, x_n]\ b = f\ x_1\ (f\ x_2\ (\ldots (f\ x_n\ b)\ldots))$$

Example: Knight Tours

Theory File: `KnightTour.thy`





Background

- Hamiltonian paths
- tree search (depth first, breadth first)

Ad

Isabelle in Innsbruck:

- IsaFoR project: **Isa***belle* **F***ormalization* **o***f* **R***ewriting* ($\sim$180,000 LOC)
- bachelor projects
- master projects
- PhD theses