



Homework

1. Using the basic HOL rules derive:

- the symmetry of equality:

$$A = B \vdash B = A$$

in two ways, both *different* from the one presented in the lecture. Show the intermediate steps.

2. Propose the HOL encodings of the basic logical connectives disjunction and negation. Can you propose multiple ways?

3. Give a closed $\lambda \rightarrow$ -term that has type $(A \rightarrow B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow A \rightarrow C$.

Can you give another such term?

Same question but now with the further restriction that the term may not contain β -redexes.

4. For each of the following λ -terms give its principal type (if it exists):

- (a) $\lambda xy.y$;
- (b) $\lambda xy.xy$;
- (c) $\lambda x.xx$;
- (d) $\lambda xy.x(xy)$;
- (e) $(\lambda xy.x(xy))(\lambda uv.u(uv))$;
- (f) $(\lambda xy.x(xy))(\lambda xy.x(xy))$;

5. Give a type that is more general than both $A \rightarrow B \rightarrow A$ and $C \rightarrow C$.

Can you give another such type?

And another one?

6. Give a derivation (if it exists) that corresponds to:

- (a) $((A \rightarrow B) \rightarrow C) \rightarrow B \rightarrow C$;
- (b) $((B \rightarrow A) \rightarrow C) \rightarrow B \rightarrow C$;
- (c) $((B \rightarrow C) \rightarrow A) \rightarrow B \rightarrow C \rightarrow B \rightarrow C$.