

Homework

1. Consider

$$\mathbb{N} = \{0^{\mathbb{N}}, S^{\mathbb{N} \rightarrow \mathbb{N}}\}$$

$$\mathbb{B} = \{T^{\mathbb{B}}, F^{\mathbb{B}}\}$$

$R_{\mathbb{N}}^r$ as in the slides

define a function $eq^{\mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{B}}$ which computes equality of such two natural numbers as a boolean using R in Gödels system T^- .

2. Given the totality predicate on \mathbb{N}

$$T(0), \forall_n^{nc} (Tn \rightarrow T(Sn))$$

And the weak elimination axiom:

$$\forall_n^{nc} (Tn \rightarrow P0 \rightarrow \forall_n^{nc} (Pn \rightarrow P(Sn)) \rightarrow Pn)$$

Find a suitable instantiation for P to prove the regular (stronger) elimination axiom:

$$\forall_n^{nc} (Tn \rightarrow Q0 \rightarrow \forall_n^{nc} (Tn \rightarrow Qn \rightarrow Q(Sn)) \rightarrow Qn)$$

3. Implement a function which given an iterator returns a recursor. An Iterator is:

$$I : \mathbb{N} \rightarrow \tau \rightarrow (\tau \rightarrow \tau) \rightarrow \tau$$

With the reductions

$$I 0 m n \Longrightarrow M$$

$$I (Sn) M N \Longrightarrow N (I n M N)$$