Logic Programming

Georg Moser

Department of Computer Science @ UIBK

Winter 2016

Organisation

Time and Place

Lecture	Monday, 10:15–11:45, HS 11	Georg Moser
Proseminar	Friday, 15:15–17:00, HS 11	(every other week)

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Schedule

week 1	October 3	week 8	November 21
week 2	October 10	week 9	November 28
week 3	October 17	week 10	December 5
week 4	October 24	week 11	December 12
week 5	October 31	week 12	January 9
week 6	November 7	week 13	January 16
week 7	November 14	week 14	January 23
		first exam	January 30

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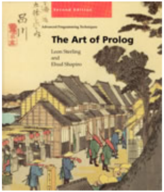
Office Hours

- Thursday, 9:00–11:00, 1N05, IfI Building

Literature

1. Leon Sterling and Ehud Shapiro
   The Art of Prolog



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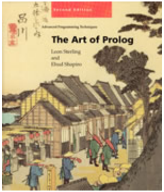

Additional Reading

- Patrick Blackburn, Johan Bos and Kristina Striegnitz
  Learn Prolog Now!
- William F. Clocksin and Christopher S. Mellish
  Programming in Prolog
- Thom Frühwirth et al.
  Essentials of Constraint Programming
- Martin Gebser et al.
  Answer Set Solving in Practice

Evaluations

Exam

- first exam will take place on January 30
- closed-book (no materials, easier questions)

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Proseminar

- lecture and proseminar are on Monday and Friday, respectively
- each weak I'll assign 3 exercises
- selection of exercises will be discussed every other week, starting October 10
- your mark depends on your level of activity in the laboratory
- exercises will be easy and few, so that everybody can solve all exercises

SWI-Prolog

```
[zid-gpl.uibk.ac.at] swipl
  Welcome to SWI-Prolog (Multi-threaded, 64 bits, Version 5.7.11)
  Copyright (c) 1990-2009 University of Amsterdam.
  SWI-Prolog comes with ABSOLUTELY NO WARRANTY. This is free software,
  and you are welcome to redistribute it under certain conditions.
  Please visit http://www.swi-prolog.org for details.

  For help, use ?- help(Topic). or ?- apropos(Word).

  ?-
```

Emacs Mode

Bruda's Prolog Mode

1. goto `http://bruda.ca/emacs/prolog_mode_for_emacs`
2. download prolog.el, compile and put into sub-directory site-lisp
3. put the following into .emacs:

```
(autoload 'run-prolog "prolog"
          "Start a Prolog sub-process." t)
(autoload 'prolog-mode "prolog"
           "Major mode for editing Prolog programs." t)
(setq prolog-system 'swi)
(setq auto-mode-alist
      (cons (cons "\\.pl" 'prolog-mode) auto-mode-alist))
```

Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, logic foundations, unification, semantics, database and recursive programming, termination, complexity

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

Full Prolog

semantics (revisted), cuts, correctness proofs, meta-logical predicates, efficient programs, meta programming

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Logic Programs

Attempt at a Definition

logic programming is a declarative programming paradigm, that is, the specification of a problem is made a first-class citizen; the idea can be summarised as follows:

program	set of judgements
computation	proof of a goal statement from the program

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Advertisement

> *In its ultimate and purest form, logic programming suggests that even explicit instructions for operations not be given, but, rather, the knowledge about the problem and assumptions that are sufficient to solve it be stated explicitly, as logical axioms.*

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this is very abstract, over-simplified, and becomes false, when subject to scrutiny ... still logic programming is a pearl

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A Few Applications

- speech recognition: Clarissa
- networks: Ericsson Network Resource Manager
- program analysis: Julia, CoFloCo

Basic Constructs

Definitions

- terms are built from logical variables, constants and functors
- ground term contains no variables; nonground term contains variables

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Notation

we confuse function symbols and predicate symbols (= functors) in the definition of a term; this makes meta-level predicates more natural

Example (Goal)

```
father(andreas, boris)
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Definitions (Clause)

- a clause or rule is an universally quantified logical formula of the form

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  A :- B1, B2, ..., Bn.
  ```

  where A and the B_i's are goals
- A is called the head of the clause; the B_i's are called the body
- a rule of the form `A :-` is called a fact; we write facts simply A.

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Definition

a logic program is a finite set of clauses

Example (Facts)

```
father(andreas, boris).
father(andreas, christian).
father(andreas, doris).
father(boris, eva).
father(franz, georg).
mother(helga, doris).
mother(doris, franz).
mother(anna, eva).
mother(eva, georg).

female(doris).
female(eva).

male(andreas).
male(boris).
male(christian).
male(franz).
male(georg).
```

Example (Facts)

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father(andreas, boris).        female(doris).  male(andreas).
father(andreas, christian).    female(eva).    male(boris).
father(andreas, doris).                        male(christian).
father(boris, eva).                            male(franz).
father(franz, georg).                          male(georg).
mother(helga, doris).
mother(doris, franz).
mother(anna, eva).
mother(eva, georg).
```

Example (Rules)

```
daughter(X,Y) :- father(Y,X), female(X).
daughter(X,Y) :- mother(Y,X), female(X).
grandfather(X,Y) :- father(X,Z), father(Z,Y).
grandfather(X,Y) :- father(X,Z), mother(Z,Y).
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Definition (Queries and Use Cases)

a complex query is a conjunction of goals of the following form:

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:- A1, A2, ..., An
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Observations

1. existential query contains logical variable(s)

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Observations

1. existential query contains logical variable(s)
2. universal fact contains logical variable(s)
3. conjunctive query is conjunction of goals posed as query
4. it is good style to write use case before the actual program

Definitions

- substitution is finite set of pairs

$$\{X_1 \mapsto t_1, \ldots, X_n \mapsto t_n\}$$

with terms $t_1, \ldots, t_n$ and pairwise different variables $X_1, \ldots, X_n$

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Examples

$$\theta_1 = \{\mathtt{X} \mapsto \mathtt{boris}\}$$
$$\theta_2 = \{\mathtt{X} \mapsto \mathtt{boris}, \mathtt{Y} \mapsto \mathtt{eva}\}$$
$$\theta_3 = \{\mathtt{X} \mapsto \mathtt{s(Y)}, \mathtt{Y} \mapsto \mathtt{0}\}$$

$$\mathtt{father(andreas, X)}\theta_1 = \mathtt{father(andreas, boris)}$$
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Example (Addition on Natural Numbers)

```
natural_number(0).
natural_number(s(X)) :- natural_number(X).

plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
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Queries

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$X = 0$, $Y = 0$

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X = s(s(0)), Y = s(s(s(s(0))))

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:- times(X,X,Y).

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X = s(s(0)), Y = s(s(s(s(0)))) ;

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:- plus(X,s(0),0).

false

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Queries

:- times(X,X,Y).
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X = s(0), Y = s(0) ;
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:- plus(X,s(0),0).

false

:- plus(X,s(0),s(s(X))).

:- plus(s(0),X,s(s(X))).

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Queries

:- times(X,X,Y).	:- plus(X,s(0),0).
$X = 0$, $Y = 0$;	false
$X = s(0)$, $Y = s(0)$;	:- plus(X,s(0),s(s(X))).
$X = s(s(0))$, $Y = s(s(s(s(0))))$;	:- plus(s(0),X,s(s(X))).

Demo

SWI-Prolog

Comparison to Conventional Programming Languages

Fact

a programming language is characterised by its control and data manipulation mechanisms

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Control

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A :- B1, B2, ..., Bn
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procedure A
call $B1$
call $B2$
⋮
call Bn
end

Comparison to Conventional Programming Languages

Fact

a programming language is characterised by its control and data manipulation mechanisms

Control

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procedure A
    call $B1$
    call $B2$
    ⋮
    call Bn
end

Observations

1. goal invocation corresponds to procedure invocation
2. differences show when backtracking occurs

Data Structures

1. data structures manipulated by logic programs (= terms) correspond to general record structures
2. like LISP, Prolog is a declaration free, untyped language
3. Prolog does not support destructive assignment where the content of the initialised variable can change

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Data Manipulation

1. data manipulation is achieved via unification
2. unification subsumes
   - single assignment
   - parameter passing
   - record allocation
   - read/write-once field access in records