

# Logic Programming

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### Summary of Last Lecture

```
Example
```

```
ancestor_of(Ancestor, Descendant) :-
    child_of(Descendant, Ancestor).
ancestor_of(Ancestor, Descendant) :-
    child_of(Person, Ancestor),
    ancestor_of(Person, Descendant).
```

#### Example

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ancestor_of_2(Ancestor, Descendant) :-
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ancestor_of_2(Ancestor, Descendant) :-
    ancestor_of_2(Person, Descendant),
    child_of(Person, Ancestor).
```

## Outline of the Lecture

### Monotone Logic Programs

introduction, basic constructs, logic foundations, unification, semantics, database and recursive programming, termination, complexity

#### Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

#### Full Prolog

semantics (revisted), correctness proofs, meta-logical predicates, cuts nondeterministic programming, efficient programs, complexity

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### Non-Monotonic Reasoning

#### Definition

#### an operator $\Psi$ is called monotone if $A \subseteq B$ implies $\Psi(A) \subseteq \Psi(B)$

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an operator  $\Psi$  is called monotone if  $A \subseteq B$  implies  $\Psi(A) \subseteq \Psi(B)$ 

#### Fact

suppose  $\Psi$  acts on sets of formulas and interprets the consequence relation of a logic program P without negation, then  $\Psi$  is monotone

Definition

a monotone logic program is a logic program without negation (+)

## Monotonicity Criticism

```
Example (Minsky's Example)
on(a,b).
on(d,a).
on(d,c).
clear(Y) :-
    not_exists_x_on(Y).
not_exists_x_on(Y) :-
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on(X,Y), !, fail.
not_exists_x_on(Y).
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not_exists_x_on(Y) :-
    on(_X,Y), !, fail.
not_exists_x_on(_Y).
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### Observations

- in this block-world example :- clear(d) holds
- but monotoncity doesn't; addition of the fact on(e,d). renders
  - $:= \ \mathsf{clear} \left( \mathsf{d} \right) \ false$

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- resolvent of goal clause :- B<sub>1</sub>,..., B<sub>i</sub>,..., B<sub>m</sub> and rule
   A :- A<sub>1</sub>,..., A<sub>n</sub>
   is goal clause

$$:= B_1\sigma, \ldots, B_{i-1}\sigma, A_1\sigma, \ldots, A_n\sigma, B_{i+1}\sigma \ldots, B_m\sigma$$

provided  $B_i$  (selected goal) and A unify with most general unifier  $\sigma$ 

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NB: see week 2 for the most general unifier

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- SLD refutation is finite SLD derivation ending in  $\square$
- computed answer substitution of SLD refutation of P and G with substitutions  $\sigma_0, \sigma_1, \ldots, \sigma_m$  is restriction of  $\sigma_0 \sigma_1 \cdots \sigma_m$  to variables in G

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
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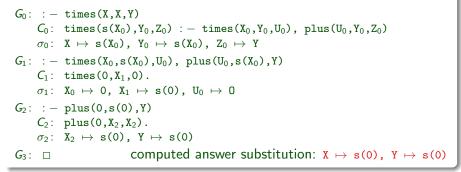
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### Theorem

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such that  $\sigma'$  is at least as general as  $\sigma$  (with respect to variables in G)

### Search or SLD Trees

Definition

a search tree (aka SLD tree) of a goal G is a tree T such that

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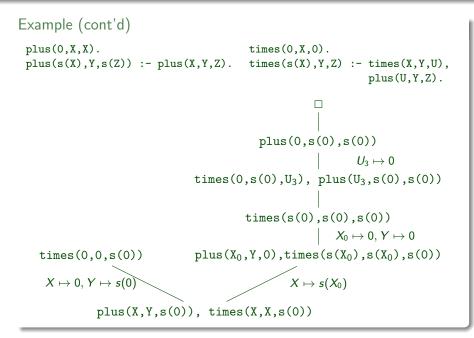
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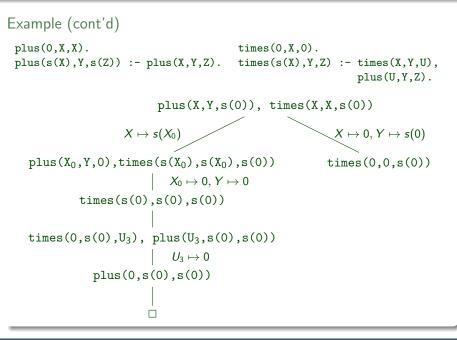
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## Remark

a search tree captures all possible SLD derivations wrt a given goal and selection function





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## In English

Someone is ancestor of a descendant, if the descendant is his (or her) child, or if he (or she) has a child and this person is the ancestor of the descendant.

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  - if the specialisation provides wrong answers, the original program certainly will

### 2 generalisation

- if we remove goals from the body of a clause, the relation is extended; the program is generalised
- if the generalised program cannot derive correct facts, the original can neither

```
Example (multiplication) logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

```
:- plus(s(s(0)),s(0),s(s(s(0))))
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:= plus(s(s(0)), s(0), s(s(s(0)))) \quad X \mapsto s(0), \ Y \mapsto s(0), \ Z \mapsto s(s(0))
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- :- plus(s(s(0)),s(0),s(s(s(0))))  $X \mapsto s(0), Y \mapsto s(0), Z \mapsto s(s(0))$
- :- plus(s(0),s(0),s(s(0)))

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X\mapsto 0, Y\mapsto s(0), Z\mapsto s(0)
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### ground queries

- :- plus(s(s(0)),s(0),s(s(s(0))))
- :- plus(s(0),s(0),s(s(0)))

 $X\mapsto 0$ ,  $Y\mapsto s(0)$ ,  $Z\mapsto s(0)$ 

:- plus(0,s(0),s(0))

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Example (multiplication) logic program
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- :- plus(s(s(0)),s(0),s(s(s(0))))
- :- plus(s(0),s(0),s(s(0)))
- $:= plus(0, s(0), s(0)) \qquad \qquad X \mapsto s(0)$

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plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
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times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

### ground queries

- :- plus(s(s(0)),s(0),s(s(s(0))))
- :- plus(s(0),s(0),s(s(0)))
- :- plus(0,s(0),s(0))

#### solved

```
... is Too Complicated
```

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

```
:- plus(s(s(0)),s(0),X)
```

```
... is Too Complicated
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Example (renaming is needed)
logic program
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```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

```
:- plus(s(s(0)),s(0),X)
```

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X<sub>1</sub>),Y<sub>1</sub>,s(Z<sub>1</sub>)) : - plus(X<sub>1</sub>,Y<sub>1</sub>,Z<sub>1</sub>).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

```
:= plus(s(s(0)), s(0), X) \qquad X_1 \mapsto s(0), \ Y_1 \mapsto s(0), \ X \mapsto s(Z_1)
```

```
Example (renaming is needed)
logic program
```

```
\begin{array}{l} plus(0,X,X).\\ plus(s(X_1),Y_1,s(Z_1)) \ :\ - \ plus(X_1,Y_1,Z_1).\\ times(0,X,0).\\ times(s(X),Y,Z) \ :\ - \ times(X,Y,U), \ plus(U,Y,Z). \end{array}
```

#### query

- :- plus(s(s(0)),s(0),X)
- :-  $plus(s(0), s(0), Z_1)$

 $X \mapsto s(Z_1)$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

- :- plus(s(s(0)),s(0),X)
- :-  $plus(s(0), s(0), Z_1)$

 $X \mapsto s(Z_1)$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X<sub>2</sub>),Y<sub>2</sub>,s(Z<sub>2</sub>)) :- plus(X<sub>2</sub>,Y<sub>2</sub>,Z<sub>2</sub>).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

#### query

 $\begin{array}{ll} :- \operatorname{plus}(s(s(0)), s(0), X) & X \mapsto s(Z_1) \\ :- \operatorname{plus}(s(0), s(0), Z_1) & X_2 \mapsto 0, \ Y_2 \mapsto s(0), \ Z_1 \mapsto s(Z_2) \end{array}$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X<sub>2</sub>),Y<sub>2</sub>,s(Z<sub>2</sub>)) : - plus(X<sub>2</sub>,Y<sub>2</sub>,Z<sub>2</sub>).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

- :- plus(s(s(0)),s(0),X)
- :- plus(s(0),s(0),Z<sub>1</sub>)
- :-  $plus(0, s(0), Z_2)$

 $X\mapsto s(Z_1)\ Z_1\mapsto s(Z_2)$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

- :- plus(s(s(0)),s(0),X)
- :- plus(s(0),s(0),Z<sub>1</sub>)
- :-  $plus(0, s(0), Z_2)$

 $egin{aligned} X &\mapsto s(Z_1) \ Z_1 &\mapsto s(Z_2) \end{aligned}$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X<sub>3</sub>,X<sub>3</sub>).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

 $\begin{array}{ll} :- \ \text{plus}(s(s(0)), s(0), X) & X \mapsto s(Z_1) \\ :- \ \text{plus}(s(0), s(0), Z_1) & Z_1 \mapsto s(Z_2) \\ :- \ \text{plus}(0, s(0), Z_2) & X_3 \mapsto s(0), \ Z_2 \mapsto s(0) \end{array}$ 

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

- :- plus(s(s(0)),s(0),X)
  :- plus(s(0),s(0),Z<sub>1</sub>)
- $p_{100}(S(0), S(0), Z)$
- :-  $plus(0, s(0), Z_2)$

 $egin{aligned} X &\mapsto s(Z_1) \ Z_1 &\mapsto s(Z_2) \ Z_2 &\mapsto s(0) \end{aligned}$ 

### solution

```
Example (renaming is needed)
logic program
```

```
plus(0,X,X).
plus(s(X),Y,s(Z)) : - plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) : - times(X,Y,U), plus(U,Y,Z).
```

#### query

 $\begin{array}{ll} :- \operatorname{plus}(\mathfrak{s}(\mathfrak{s}(0)),\mathfrak{s}(0),\mathfrak{X}) & X \mapsto \mathfrak{s}(Z_1) \\ :- \operatorname{plus}(\mathfrak{s}(0),\mathfrak{s}(0),Z_1) & Z_1 \mapsto \mathfrak{s}(Z_2) \\ :- \operatorname{plus}(0,\mathfrak{s}(0),Z_2) & Z_2 \mapsto \mathfrak{s}(0) \end{array}$ 

## solution $X \mapsto s(s(s(0)))$

## Definition

- a type is a (possible infinite) set of terms
- types are conveniently defined by unary relations

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Example

male(X). female(X).

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#### Example

male(X). female(X).

#### Definition

- to define complex types, recursive logic programs may be necessary
- the latter types are called recursive types
- recursive types, defined by unary recursive programs, are called simple recursive types
- a program defining a type is a type definition; a call to a predicate defining a type is a type condition

```
Simple Recursive Types
Example
is_tree(nil).
is_tree(tree(Element,Left,Right)) : -
    is_tree(Left),
    is_tree(Right).
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is_tree(nil).
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- a type is complete if closed under the instance relation
- with every complete type *T* one associates an incomplete type *IT* which is a set of terms with instances in *T* and instances not in *T*

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Simple Recursive Types
Example
is_tree(nil).
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- a type is complete if closed under the instance relation
- with every complete type *T* one associates an incomplete type *IT* which is a set of terms with instances in *T* and instances not in *T*

#### Example

- the type  $\{0, s(0), s(s(0)), \dots\}$  is complete
- the type  $\{X, 0, s(0), s(s(0)), \dots\}$  is incomplete

# Lists Notation

• [] empty list

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- [H|T] list with head H and tail T

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- [*A*, *B*] [*A*|[*B*|[]]] list with two elements

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#### Example

is\_list([]). is\_list([X|Xs]) :- is\_list(Xs).

### Notation

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#### Example

is\_list([]). is\_list([X|Xs]) :- is\_list(Xs).

#### Notation

formal object cons pair syntax element syntax .(a,[]) [a|[]] [a] .(a,.(b,[])) [a|[b|[]]] [a,b]