Logic Programming

Georg Moser

Department of Computer Science @ UIBK

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Summary of Last Lecture

Example

```
ancestor_of(Ancestor, Descendant) :-
    child_of(Descendant, Ancestor).
ancestor_of(Ancestor, Descendant) :-
    child_of(Person, Ancestor),
    ancestor_of(Person, Descendant).
```

Example

```
ancestor_of_2(Ancestor, Descendant) :-
    child_of(Descendant, Ancestor).
ancestor_of_2(Ancestor, Descendant) :-
    ancestor_of_2(Person, Descendant),
    child_of(Person, Ancestor).
```

Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, logic foundations, unification, semantics, database and recursive programming, termination, complexity

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

Full Prolog

semantics (revisted), correctness proofs, meta-logical predicates, cuts non-deterministic programming, efficient programs, complexity

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Non-Monotonic Reasoning

Definition

an operator Ψ is called monotone if $A \subseteq B$ implies $\Psi(A) \subseteq \Psi(B)$

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an operator Ψ is called monotone if $A \subseteq B$ implies $\Psi(A) \subseteq \Psi(B)$

Fact

suppose Ψ acts on sets of formulas and interprets the consequence relation of a logic program P without negation, then Ψ is monotone

Definition

a monotone logic program is a logic program without negation (\+)

Monotonicity Criticism

Example (Minsky's Example)

```
on(a,b).
on(d,a).
on(d,c).
clear(Y) :-
    not_exists_x_on(Y).

not_exists_x_on(Y) :-
    on(_X,Y), !, fail.
not_exists_x_on(_Y).
```

Monotonicity Criticism

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Observations

- in this block-world example `:- clear(d)` holds
- but monotoncity doesn't; addition of the fact `on(e,d).` renders `:- clear(d)` false

Theory of Monotone Logic Programs

Definitions

- goal clause

$$:\!-\ B_1, \ldots, B_n$$

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- empty goal clause $:\!-$ is denoted by $\Box$
- resolvent of goal clause $:\!-\ B_1, \ldots, B_i, \ldots, B_m$ and rule $A\ :\!-\ A_1, \ldots, A_n$
  is goal clause

$$:\!-\ B_1\sigma, \ldots, B_{i-1}\sigma, A_1\sigma, \ldots, A_n\sigma, B_{i+1}\sigma \ldots, B_m\sigma$$

  provided B_i (selected goal) and A unify with most general unifier σ

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  $$:- \ B_1\sigma, \ldots, B_{i-1}\sigma, A_1\sigma, \ldots, A_n\sigma, B_{i+1}\sigma \ldots, B_m\sigma$$
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NB: see week 2 for the most general unifier

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  - G_{i+1} is resolvent of G_i and C_i with mgu σ_i
  - C_i has no variables in common with $G, C_0, \ldots, C_{i-1}$
- SLD refutation is finite SLD derivation ending in $\square$
- computed answer substitution of SLD refutation of P and G with substitutions $\sigma_0, \sigma_1, \ldots, \sigma_m$ is restriction of $\sigma_0\sigma_1 \cdots \sigma_m$ to variables in G

Example

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).

:- times(X,X,Y)
```

Example

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plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
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```

```
:- times(X,X,Y)
```

SLD-refutation

G_0: :- times(X,X,Y)

C_0: times(s(X_0),Y_0,Z_0) :- times(X_0,Y_0,U_0), plus(U_0,Y_0,Z_0)

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plus(0,X,X).
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:- times(X,X,Y)
```

SLD-refutation

G_0: $:-$ `times(X,X,Y)`

C_0: $\texttt{times(s(X}_0\texttt{),Y}_0\texttt{,Z}_0\texttt{)} :- \texttt{times(X}_0\texttt{,Y}_0\texttt{,U}_0\texttt{), plus(U}_0\texttt{,Y}_0\texttt{,Z}_0\texttt{)}$

σ_0: $\texttt{X} \mapsto \texttt{s(X}_0\texttt{)},\ \texttt{Y}_0 \mapsto \texttt{s(X}_0\texttt{)},\ \texttt{Z}_0 \mapsto \texttt{Y}$

Example

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G_0: $:-$ times(X,X,Y)

C_0: times(s(X_0),Y_0,Z_0) $:-$ times(X_0,Y_0,U_0), plus(U_0,Y_0,Z_0)

σ_0: X $\mapsto$ s(X_0), Y_0 $\mapsto$ s(X_0), Z_0 $\mapsto$ Y

G_1: $:-$ times(X_0,s(X_0),U_0), plus(U_0,s(X_0),Y)

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G_1: $:-$ times(X_0,s(X_0),U_0), plus(U_0,s(X_0),Y)

C_1: times(0,X_1,0).

σ_1: X_0 $\mapsto$ 0, X_1 $\mapsto$ s(0), U_0 $\mapsto$ 0

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σ_0: X $\mapsto$ s(X_0), $Y_0 \mapsto$ s(X_0), $Z_0 \mapsto$ Y

G_1: $:-$ times(X_0,s(X_0),U_0), plus(U_0,s(X_0),Y)

C_1: times(0,X_1,0).

σ_1: $X_0 \mapsto$ 0, $X_1 \mapsto$ s(0), $U_0 \mapsto$ 0

G_2: $:-$ plus(0,s(0),Y)

C_2: plus(0,X_2,X_2).

σ_2: $X_2 \mapsto$ s(0), Y $\mapsto$ s(0)

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G_2: :− plus(0,s(0),Y)

  C_2: plus(0,X_2,X_2).

  σ_2: X_2 $\mapsto$ s(0), Y $\mapsto$ s(0)

G_3: $\square$

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C_0: times(s(X_0),Y_0,Z_0) $:-$ times(X_0,Y_0,U_0), plus(U_0,Y_0,Z_0)

σ_0: $X \mapsto s(X_0)$, $Y_0 \mapsto s(X_0)$, $Z_0 \mapsto Y$

G_1: $:-$ times(X_0,s(X_0),U_0), plus(U_0,s(X_0),Y)

C_1: times(0,X_1,0).

σ_1: $X_0 \mapsto 0$, $X_1 \mapsto s(0)$, $U_0 \mapsto 0$

G_2: $:-$ plus(0,s(0),Y)

C_2: plus(0,X_2,X_2).

σ_2: $X_2 \mapsto s(0)$, $Y \mapsto s(0)$

G_3: $\square$ computed answer substitution: $X \mapsto s(0)$, $Y \mapsto s(0)$

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Theorem

$\forall$ *logic programs* P *and goal clause* G
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$\forall$ *selection functions* $\mathcal{S}$
$\exists$ *computed answer substitution* σ' *using* $\mathcal{S}$

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$\forall$ *computed answer substitutions* σ
$\forall$ *selection functions* $\mathcal{S}$
$\exists$ *computed answer substitution* σ' *using* $\mathcal{S}$
such that σ' *is at least as general as* σ *(with respect to variables in* G*)*

Search or SLD Trees

Definition

a search tree (aka SLD tree) of a goal G is a tree T such that

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Remark

a search tree captures all possible SLD derivations wrt a given goal and selection function

Example (cont'd)

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plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
```

```
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U),
                   plus(U,Y,Z).
```

$\square$

`plus(0,s(0),s(0))`

$U_3 \mapsto 0$

`times(0,s(0),U₃), plus(U₃,s(0),s(0))`

`times(s(0),s(0),s(0))`

$X_0 \mapsto 0, Y \mapsto 0$

`times(0,0,s(0))` `plus(X₀,Y,0),times(s(X₀),s(X₀),s(0))`

$X \mapsto 0, Y \mapsto s(0)$ $X \mapsto s(X_0)$

`plus(X,Y,s(0)), times(X,X,s(0))`

Example (cont'd)

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```

plus(X,Y,s(0)), times(X,X,s(0))

$X \mapsto s(X_0)$ | $X \mapsto 0, Y \mapsto s(0)$

plus(X_0,Y,0),times(s(X_0),s(X_0),s(0)) | times(0,0,s(0))

$X_0 \mapsto 0, Y \mapsto 0$

times(s(0),s(0),s(0))

times(0,s(0),U_3), plus(U_3,s(0),s(0))

$U_3 \mapsto 0$

plus(0,s(0),s(0))

$\square$

Example (revisited)

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ancestor_of(Ancestor, Descendant) :-
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- Ancestor

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In English

> *Someone is ancestor of a descendant, if the descendant is his (or her) child, or if he (or she) has a child and this person is the ancestor of the descendant.*

binding of logical variables is expressed as references

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Analysis

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   - if we remove clauses of a defined relation, then this relation becomes smaller; the program is specialised
   - if the specialisation provides wrong answers, the original program certainly will
2. generalisation
   - if we remove goals from the body of a clause, the relation is extended; the program is generalised
   - if the generalised program cannot derive correct facts, the original can neither

Procedure Reading

Example (multiplication)

logic program

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plus(0,X,X).
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times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

ground queries

```
:- plus(s(s(0)),s(0),s(s(s(0))))
```

Procedure Reading

Example (multiplication)

logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

ground queries

`:- plus(s(s(0)),s(0),s(s(s(0))))` $X \mapsto s(0),\ Y \mapsto s(0),\ Z \mapsto s(s(0))$

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ground queries

`:- plus(s(s(0)),s(0),s(s(s(0))))` $X \mapsto s(0),\ Y \mapsto s(0),\ Z \mapsto s(s(0))$

`:- plus(s(0),s(0),s(s(0)))`

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:- plus(s(s(0)),s(0),s(s(s(0))))
:- plus(s(0),s(0),s(s(0)))
```

$X \mapsto 0,\ Y \mapsto s(0),\ Z \mapsto s(0)$

Procedure Reading

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logic program

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```
:- plus(s(s(0)),s(0),s(s(s(0))))
:- plus(s(0),s(0),s(s(0)))
:- plus(0,s(0),s(0))
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$X \mapsto 0, \;\; Y \mapsto s(0), \;\; Z \mapsto s(0)$

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```
:- plus(s(s(0)),s(0),s(s(s(0))))
:- plus(s(0),s(0),s(s(0)))
:- plus(0,s(0),s(0))
```

$X \mapsto s(0)$

Procedure Reading

Example (multiplication)

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plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
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ground queries

```
:- plus(s(s(0)),s(0),s(s(s(0))))
:- plus(s(0),s(0),s(s(0)))
:- plus(0,s(0),s(0))
```

solved

. . . is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

```
:- plus(s(s(0)),s(0),X)
```

...is Too Complicated

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logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

```
:- plus(s(s(0)),s(0),X)
```

…is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X1),Y1,s(Z1)) :- plus(X1,Y1,Z1).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

```
:- plus(s(s(0)),s(0),X)
```

$X_1 \mapsto s(0), \; Y_1 \mapsto s(0), \; X \mapsto s(Z_1)$

... is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X1),Y1,s(Z1)) :- plus(X1,Y1,Z1).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

```
:- plus(s(s(0)),s(0),X)
:- plus(s(0),s(0),Z1)
```

$X \mapsto s(Z_1)$

...is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $\qquad X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1)

... is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X2),Y2,s(Z2)) :- plus(X2,Y2,Z2).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $\qquad X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1) $\qquad X_2 \mapsto 0,\ Y_2 \mapsto s(0),\ Z_1 \mapsto s(Z_2)$

...is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X2),Y2,s(Z2)) :- plus(X2,Y2,Z2).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

```
:- plus(s(s(0)),s(0),X)
:- plus(s(0),s(0),Z1)
:- plus(0,s(0),Z2)
```

$X \mapsto s(Z_1)$

$Z_1 \mapsto s(Z_2)$

... is Too Complicated

Example (renaming is needed)

logic program

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plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1) $Z_1 \mapsto s(Z_2)$

:- plus(0,s(0),Z_2)

…is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X3,X3).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $\quad X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1) $\quad Z_1 \mapsto s(Z_2)$

:- plus(0,s(0),Z_2) $\quad X_3 \mapsto s(0),\ Z_2 \mapsto s(0)$

...is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :- plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :- times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1) $Z_1 \mapsto s(Z_2)$

:- plus(0,s(0),Z_2) $Z_2 \mapsto s(0)$

solution

. . . is Too Complicated

Example (renaming is needed)

logic program

```
plus(0,X,X).
plus(s(X),Y,s(Z)) :− plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) :− times(X,Y,U), plus(U,Y,Z).
```

query

:- plus(s(s(0)),s(0),X) $X \mapsto s(Z_1)$

:- plus(s(0),s(0),Z_1) $Z_1 \mapsto s(Z_2)$

:- plus(0,s(0),Z_2) $Z_2 \mapsto s(0)$

solution $X \mapsto s(s(s(0)))$

Definition

- a type is a (possible infinite) set of terms
- types are conveniently defined by unary relations

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Example

```
male(X).    female(X).
```

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male(X).    female(X).
```

Definition

- to define complex types, recursive logic programs may be necessary
- the latter types are called recursive types
- recursive types, defined by unary recursive programs, are called simple recursive types
- a program defining a type is a type definition; a call to a predicate defining a type is a type condition

Simple Recursive Types

Example

```
is_tree(nil).
is_tree(tree(Element,Left,Right)) :-
    is_tree(Left),
    is_tree(Right).
```

Simple Recursive Types

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Definition

- a type is complete if closed under the instance relation
- with every complete type T one associates an incomplete type IT which is a set of terms with instances in T and instances not in T

Simple Recursive Types

Example

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is_tree(nil).
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```

Definition

- a type is complete if closed under the instance relation
- with every complete type T one associates an incomplete type IT which is a set of terms with instances in T and instances not in T

Example

- the type $\{0, s(0), s(s(0)), \dots\}$ is complete
- the type $\{X, 0, s(0), s(s(0)), \dots\}$ is incomplete

Lists

Notation

- `[]` empty list

Lists

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- [] empty list
- $[H \mid T]$ list with head H and tail T

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- [] empty list
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Lists

Notation

[]	empty list	
$[H \mid T]$	list with head H and tail T	
$[A]$	$[A \mid []]$	list with one element
$[A, B]$	$[A \mid [B \mid []]]$	list with two elements
$[A, B \mid T]$	$[A \mid [B \mid T]]$	list with at least two elements

Lists

Notation

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- $[A, B \mid T]$ $[A \mid [B \mid T]]$ list with at least two elements

Example

```
is_list([]).   is_list([X|Xs]) :- is_list(Xs).
```

Lists

Notation

- $[]$ empty list
- $[H|T]$ list with head H and tail T
- $[A]$ $[A|[]]$ list with one element
- $[A,B]$ $[A|[B|[]]]$ list with two elements
- $[A,B|T]$ $[A|[B|T]]$ list with at least two elements

Example

```
is_list([]).  is_list([X|Xs]) :- is_list(Xs).
```

Notation

formal object	cons pair syntax	element syntax
`.(a,[])`	`[a|[]]`	`[a]`
`.(a,.(b,[]))`	`[a|[b|[]]]`	`[a,b]`