Logic Programming

Georg Moser

Department of Computer Science @ UIBK

Winter 2016

Summary of Last Lecture

Definition

the difference of two lists is denotes as $As \setminus Bs$ and called difference list

Example

```
number(I) -->
    digit(D0),
    digits(D),
    {number_codes(I,[D0|D])}.

digits([D|T]) -->
    digit(D), digits(T).

digits([]) -->
    [].
digit(D) -->
    [D],
    {code_type(D, digit)}.
```


Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, logic foundations, unification, semantics, database and recursive programming, termination, complexity

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

Full Prolog

semantics (revisted), correctness proofs, meta-logical predicates, cuts non-deterministic programming, efficient programs, complexity

Example (Definite Clause Grammars)

```
sentence(sentence(N,V)) → noun_phrase(N), verb_phrase(V).
noun_phrase(np(D,N)) → determiner(D), noun_phrase2(N).
noun_phrase(np(N)) → noun_phrase2(N).
noun_phrase2(np2(A,N)) → adjective(A), noun_phrase2(N).
noun_phrase2(np2(N)) → noun(N).
verb_phrase(vp(V,N)) → verb(V), noun_phrase(N).
verb_phrase(vp(V)) → verb(V). determiner → [the].
determiner → [a].
noun → [pie-plate].
noun → [surprise].
adjective → [decorated].
verb → [contains].
```

sentence(PT) $\overset{*}{\Rightarrow}$ "the decorated pie-plates contain a surprise"

Example

sentence(PT) $\overset{*}{\Rightarrow}$ ``the decorated pie-plate contains a surprise''
sentence(PT) $\overset{*}{\Rightarrow}$ ``the decorated pie-plates contain a surprise''

Example

```
determiner(det(the)) → [the].
determiner(det(a)) → [a].
noun(noun(pie-plate)) → [pie-plate].
noun(noun(pie-plates)) → [pie-plates].
noun(noun(surprise)) → [surprise].
noun(noun(surprises)) → [surprises].
adjective(adj(decorated)) → [decorated].
verb(verb(contains)) → [contains].
verb(verb(contain)) → [contain].
```

sentence(PT) $\overset{*}{\Rightarrow}$ ``the decorated pie-plates contains a surprise''

Extension: Number Agreement

Example

```
sentence(sentence(NP,VP),Num) →
    noun_phrase(N,Num), verb_phrase(V,Num).
⋮
determiner(det(the),Num) → [the].
determiner(det(a),singular) → [a].
noun(noun(pie-plate),singular) → [pie-plate].
noun(noun(pie-plates),plural) → [pie-plates].
noun(noun(surprise),singular) → [surprise].
noun(noun(surprises),plural) → [surprises].
adjective(adj(decorated)) → [decorated].
verb(verb(contains),singular) → [contains].
verb(verb(contain),plural) → [contain].
```

sentence(PT) $\overset{*}{\Rightarrow}$ ``the decorated pie-plates contain a surprise''

Example

```
sentence -->
    subject,
    predicate.

subject -->
    [the], [big], [bear].
subject -->
    "the", "little", "lion".

predicate -->
    [roars].
predicate -->
    [is, happy].
predicate -->
    [lives, in, the, golden, city].

:- phrase(sentence,Text), Text = [the, big, bear, roars].
:- phrase(sentence,Text), Text = [116, 104, 101, 108, 105|_].
```


Regular Predicate from Within a DCG

Task

write a DCG for number(N) that recognised numbers in English:

```
?- phrase(number(N),"onehundredandseventyfive").
N = 175 ;
```

Definition

Prolog provides an arithmetical interface

Value is *Expression*

Example

X is 3+5	8 is 3+5	N is N+1
X ↦ 8	true	*nonsensical*

Arithmetic Operations

- + - * // (integer division) / (float division)
- ...

Arithmetic Comparison Relations

- < =< > >=
- =:= (equality)
- =\= (disequality)

Non Standard Predicates

- between(*Low*,*High*,*Value*) is true when
  1. *Value* is an integer, and $Low \leqslant Value \leqslant High$
  2. *Value* is a variable, and $Value \in [Low, High]$
- succ(Int1,Int2) ...

Example (Factorials)

```
factorial(0,s(0)).

factorial(s(N),F) ←
  factorial(N,F1),
  times(s(N),F1,F).

factorial(N,F) ←
  N>0, N1 is N-1,
  factorial(N1,F1),
  F is N * F1.

factorial(0,1).
```

Example (Fibonacci Numbers)

```
fibonacci(0,1).
fibonacci(1,1).
fibonacci(N,X) :-
    N > 1,
    N1 is N-1, fibonacci(N1,Y),
    N2 is N-2, fibonacci(N2,Z),
    X is Y+Z.

?- fibonacci(3,X).
X ↦ 2
true
```


Solution

```
number(0) --> "zero".
number(N) --> xxx(N).

xxx(N) --> digit(D), "hundred", rest_xxx(N1),
           {N is D * 100 + N1}.

rest_xxx(0) --> "".
rest_xxx(N) --> "and", xx(N).

xx(N) --> digit(N).
xx(N) --> teen(N).
xx(N) --> tens(T), rest_xx(N1), {N is T + N1}.

rest_xx(0) --> "".
rest_xx(N) --> digit(N).

digit(1) --> "one".
digit(2) --> "two".
teen(10) --> "ten".
tens(20) --> "twenty".
```


Example

```
deriv_from_(X,X,1) -->
    [D],
    {atom_char(X,D)}.
deriv_from_(N,X,0) -->
    number(N).
...
deriv_from_(A+B,X,DerA+DerB) -->
    "(", deriv_from_(A,X,DerA),
    "+",
    deriv_from_(B,X,DerB), ")".
deriv_from_(A*B,X,DerA*B+A*DerB) -->
    "(", deriv_from_(A,X,DerA),
    "*",
    deriv_from_(B,X,DerB), ")".
deriv_from_(A-B,X,DerA-DerB) -->
    "(", deriv_from_(A,X,DerA),
    "-",
    deriv_from_(B,X,DerB), ")".

:- phrase(deriv_from_(A,x,ADer),`(((x*x)+y)-(3*(x+1)))`).
```

Once Again: Difference Lists

Example

```
:- listen_zusammen([[1,2],[],[4,5]],[1,2,4,5]).

listen_zusammen(Xss,Xs) :-
        phrase(seqq(Xss),Xs).

seqq([]) -->
        [].
seqq([Xs|Xss]) -->
        seq(Xs),
        seqq(Xss).

seq([]) -->
        [].
seq([C|Cs]) -->
        [C],
        seq(Cs).
```


Example (reverse revisited)

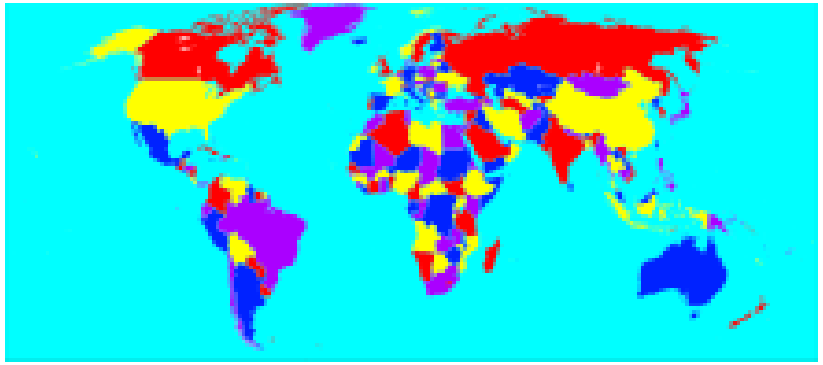
```
reverse_dl(Xs,Ys) :-
    reverse_dl(Xs,Ys,[]).

reverse_dl([],Ys,Ys).
reverse_dl([X|Xs],Ys0,Ys) :-
    reverse_dl(Xs,Ys0,[X|Ys]).

:- reverse_dl(Xs,[b,a]), Xs = [a, b].

reverse_dcg(List,Reversed) :-
    phrase(reverse(List),Reversed).

reverse([]) -->
    [].
reverse([X|Xs]) -->
    reverse(Xs),
    [X].

:- reverse_dcg(Xs,[b,a]), Xs = [a, b].
```


Generate and Test

Theorem (Four Colour Theorem)

no more than four colours are required to colour the regions of a map so that no two adjacent regions have the same color

Example



Auxiliary Predicates

Example

```
select(X,[X|Xs],Xs).
select(X,[Y|Ys],[Y|Zs]) :-
    select(X,Ys,Zs).

member(X,[X|_Xs]).
member(X,[_Y|Xs]) :-
    member(X,Xs).

:- subset_of([b,d],[a,b,c,d]).

subset_of([],_Ys).
subset_of([X|Xs],Ys) :-
    member(X,Ys),
    subset_of(Xs,Ys).
```

Generate and Test

Example

```
is_map([region(a,A,[B,C,D]), region(b,B,[A,C,E]),
    region(c,C,[A,B,D,E,F]), region(d,D,[A,C,F]),
    region(e,E,[B,C,F]), region(f,F,[C,D,E])]).

coloured_map([Region|Regions], Colours) :-
    coloured_region(Region,Colours),
    coloured_map(Regions,Colours).
coloured_map([],Colours).

coloured_region(region(Name,Colour,Neighbours), Colours) :-
    select(Colour,Colours,Colours1),
    subset_of(Neighbours,Colours1).

test_colour(Map) :-
    is_map(Map),
    is_colours(Colours),
    coloured_map(Map,Colours).
```


Nondeterministic Programming

Example

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\varnothing$	$\{q_2\}$
$*q_2$	$\varnothing$	$\varnothing$

Definition

A NFA is quintuple $(Q, \Sigma, \Delta, I, F)$ such that

1. Q is a set of states
2. Σ is an alphabet
3. Δ is relation on $(Q \times \Sigma) \times Q$
4. I are the initial states
5. F are the final states

Example

```
accept(S) :-
    initial(Q),
    accept(Q,S).

accept(Q,[X|Xs]) :-
    delta(Q,X,Q1),
    accept(Q1,Xs).
accept(Q,[]) :-
    final(Q).

initial(q0).
final(q2).

delta(q0,0,q0).
delta(q0,0,q1).
delta(q0,1,q0).
delta(q1,1,q2).

:- accept([0,0,0,1,0,1]).
```


Type Predicates

Recall

type predicates are unary relations concerning the type of a term

Definition

- `is_list`: type check for a list
- `integer`: type check for an integer
- `atom`: type check for an atom
- `compound`: type check for a compound term

Example

```
constant(X) :-
    integer(X).
constant(X) :-
    atom(X).
```

Example

```
:- flatten([[a],[b,[c,d]],e],[a,b,c,d,e]).
:- \+ listen_zusammen([[a],[b,[c,d]],e],[a,b,c,d,e]).
 flatten([X|Xs],Ys) :-
    is_list(X), flatten(X,Ys1),
    flatten(Xs,Ys2), append(Ys1,Ys2,Ys).
 flatten(X,[X]) :- constant(X), X ≠ [].
flatten([],[]).
```

Example

```
flatten(Xs,Ys) :- flatten(Xs,[],Ys).
flatten([X|Xs],Stack,Ys) :-
    is_list(X), flatten(X,[Xs|Stack],Ys).
flatten([X|Xs],Stack,[X|Ys]) :-
    constant(X), X ≠ [], flatten(Xs,Stack,Ys).
flatten([],[X|Stack],Ys) :- flatten(X,Stack,Ys).
flatten([],[],[]).
```