Logic Programming

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Summary of Last Lecture

Definitions (CLP on finite domains)

- `use_module(library(clpfd))` loads the clpfd library
- `Xs ins N .. M` specifies that all values in *Xs* must be in the given range
- `all_different(Xs)` specifies that all values in *Xs* are different
- `label(Xs)` all variables in *Xs* are evaluated to become values
- `#=`, `#\=`, `#>`, ... like the arithmetic comparison operators, but may contain (constraint) variables

standard approach

- load the library
- specify all constraints
- call `label` to start efficient computation of solutions

Outline of the Lecture

Monotone Logic Programs

introduction, basic constructs, logic foundations, unification, semantics, database and recursive programming, termination, complexity

Incomplete Data Structures and Constraints

incomplete data structures, definite clause grammars, constraint logic programming, answer set programming

Full Prolog

semantics (revisted), correctness proofs, meta-logical predicates, cuts non-deterministic programming, efficient programs, complexity

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Performance

"Neng-Fa Zhou, the author of B-Prolog, has kindly integrated our constraint solver in his benchmarks, available from `http://www.probp.com/performance.htm`. The results show that our solver is on average two orders of magnitude slower on these benchmarks than the fastest system (B-Prolog itself), and about 30 times slower than the constraint solver of SICStus Prolog."[1]

[1] Markus Triska: The Finite Domain Constraint Solver of SWI-Prolog. FLOPS 2012: 307-316

Example

a $n \times n$ square is magic if cells contain $\{1, \ldots, n^2\}$ and the row sums, the column sums and the sums of both diagonals are all equal

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Example

```
magicsquare3(Xs) :-
    magicsquare3_(Xs,Ys),
    labeling([],Ys).

magicsquare3_([[X1,X2,X3],[X4,X5,X6],[X7,X8,X9]],Ys) :-
    Ys = [X1,X2,X3,X4,X5,X6,X7,X8,X9],
    Ys ins 1..9,
    all_different(Ys),
    X1 + X2 + X3 #= N, X4 + X5 + X6 #= N,
    X7 + X8 + X9 #= N,
    X1 + X4 + X7 #= N, X2 + X5 + X8 #= N,
    X3 + X6 + X9 #= N,
    X1 + X5 + X9 #= N, X7 + X5 + X3 #= N.
```

Example

```
/* magic square + bit math:  N = n*(n^2+1)/2 */

magicsquare3_2(Xs) :-
    magicsquare3_(Xs,Ys),
    labeling([],Ys).

magicsquare3_2_([[X1,X2,X3],[X4,X5,X6],[X7,X8,X9]],Ys) :-
    Ys = [X1,X2,X3,X4,X5,X6,X7,X8,X9],
    Ys ins 1..9,
    all_different(Ys),
    N #= 15,
    X1 + X2 + X3 #= N, X4 + X5 + X6 #= N,
    X7 + X8 + X9 #= N,
    X1 + X4 + X7 #= N, X2 + X5 + X8 #= N,
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```

Example

remove symmetric solutions, due to rotations and mirroring

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```
magicsquare3nred(Xs) :-
    magicsquare3nred_(Xs,Ys),
    labeling([],Ys).

magicsquare3nred_(Xs,Ys) :-
    magicsquare3_(Xs,Ys),
    Ys = [X1,X2,X3,X4,_X5,X6,_X7,X8,X9],
    X1 #> X3,
    X6 #> X9,
    X2 #> X4,
    X6 #> X8.

:- time(magicsquare3nred(_Xs)).
% 177,052 inferences, 0.060 CPU in 0.060 seconds
% proper testing shows even speed up over clever variant
```

Efficient Constraint Logic Programmming

Strategies for Solutions

- take termination seriously
  non-termination is a sign of inefficiency
- choose suitable labeling strategies
- use system predicates

```
:- Zs = [A,B,C], Zs ins 1..2,
   A #\= B, B #\= C, A #\= C.
:/- Zs = [A,B,C], Zs ins 1..2,
    all_different(Zs).
```

- make use of redundant constraints
  recall the magic square example, where the sums equal $n \cdot (n^2 + 1)/2$; using this redundant constraint, the search may be quicker; however, such constraints are difficult to find

Labeling Strategies

Strategies for Solutions (cont'd)

- minimise the solution space
  consider the exclusion of rotations and symmetries for magic square
- improve representation of solutions
  inefficient/redundant representations increase the solution space unnecessarily

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Definition

labeling (+Options,+Vars) assign a value to each variable in *Vars*; three categories of options exist

- variable selection strategy
- value order strategy
- branching strategy

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```
:- X in 1..2, Y in 3..4, labeling([min],[X,Y]).
X = 1, Y = 3;
X = 1, Y = 4;
X = 2, Y = 3;
X = 2, Y = 4
```

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- ff, first fail, select the leftmost variable with smallest domain next, in order to detect infeasibility early

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Definition (branching strategy)

- step, for each variable X, the choice is between $X = V$ and X #\= V (V determined by value order)
- enum, enumerate the domain of X according to the value order
- bisect, choice is between X #=< M and X #> M (M the midpoint of the domain)

The New Kid on the Block

Answer Set Programming

- novel approach to modelling and solving search and optimisation problems
- $\neg$ programming, but a specification language
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- purely declarative
- restricted to finite models

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Example (Negation as Failure)

```
light_on :- power_on, not broken.
power_on.
```

answer set: $\{power\_on, light\_on\}$

Example (Disjunctive Heads)

```
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```
a | b.
a :- b.
```

answer set: $\{a\}$, but not $\{b\}$ nor $\{a, b\}$

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any answer set must not contain *b* and constraint simplifies to

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Additional Features

- finite choice functions: $\{fact_1, fact_2, fact_3\}$.
- choice and counting: $1\{fact_1, fact_2, fact_3\}2$.
  "1" or "2" may be missing

First-Order Setting

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Example (3-colouring)

```
red(X) | green(X) | blue(X).
:- red(X), red(Y), edge(X,Y).
:- green(X), green(Y), edge(X,Y).
:- blue(X), blue(Y), edge(X,Y).
```

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:- green(X), green(Y), edge(X,Y).
:- blue(X), blue(Y), edge(X,Y).
```

Example ((part of) 8-queens problem)

```
:- row(X), not (1 = count(Y : queen(X,Y)))
```

expresses that exactly one queen appears in every row and column

Grounders and Solvers

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Grounders

- DLV (DLV Systems, Calabria)
- Gringo (University of Potsdam)
- lparse (University of Helsinki)

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Solvers

- clasp (University of Potsdam)
- cmodels (University of Austin)
- smodels (University of Helsinki)

Prolog and Answer Set Programming

- proof search
- Turing complete
- control
- efficiency

- model search
- finite domain
- specification language
- generality

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Example

```
hanoi(0,_,_,_,[]).
hanoi(N,X,Y,Z,Ls) :-
      N > 0, M is N - 1,
      hanoi(M,X,Z,Y,Ls0),
      append(Ls0,[move(N,X,Z)],Ls1),
      hanoi(M,Y,X,Z,Ls2),
      append(Ls1,Ls2,Ls).
```

Example

```
disk(1..n).                              peg(a;b;c).
transition(0..pathlength-1).             situation(0..pathlength).
location(Peg) :- peg(Peg).               location(Disk) :- disk(Disk).
#domain disk(X;Y).       #domain peg(P;P1;P2).
#domain transition(T). #domain situation(I).
#domain location(L;L1).

on(X,L,T+1) :- on(X,L,T), not otherloc(X,L,T+1).
otherloc(X,L,I) :- on(X,L1,I), L1!=L.
:- on(X,L,I), on(X,L1,I), L!=L1.
inpeg(X,P,I) :- on(X,L,I), inpeg(L,P,I).    inpeg(P,P,I).
top(P,L,I) :- inpeg(L,P,I), not covered(L,I).
covered(L,I) :- on(X,L,I).
:- on(X,Y,I), X>Y.
on(X,L,T+1) :- move(P1,P2,T), top(P1,X,T), top(P2,L,T).
:- move(P1,P2,T), top(P1,P1,T). movement(P1,P2) :- P1 != P2.
1 {move(A,B,T) : movement(A,B) } 1.
on(n,a,0).                               on(X,X+1,0) :- X<n.
onewrong :- not inpeg(X,c,pathlength).
:- onewrong.
```