

Functional Programming

Christian Sternagel Harald Zankl Evgeny Zuenko

Department of Computer Science University of Innsbruck

WS 2017/2018



Organization

- LV-Number 703024
- VO 2
- http://cl-informatik.uibk.ac.at/teaching/ws17/fp/
- slides are also available online
- office hours: Friday 14:15-15:45 in 3M03
- online registration required before 23:59 on November 30

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- slides are also available online
- office hours: Friday 14:15 15:45 in 3M03
- online registration required before 23:59 on November 30
- grading: written exam (closed book)
 - 1st exam on February 2, 2018
 - registration starts 5 weeks before exam
 - registration closes 2 weeks before exam

- LV-Number 703025
- PS 1
- group 1 Christian Sternagel group 2 Harald Zankl group 3 Harald Zankl group 4 Evgeny Zuenko
 Friday 10:15-11:00 HS 11 Friday 11:15-12:00 HS 11 Friday 12:15-13:00 HS 11 Friday 13:15-14:00 HS 11
- online registration required before 23:59 on September 21
- grading: 1 test (January 12, 2018) + weekly exercises
- exercises start on October 20

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Schedule

week 1	October	6	week 8	December	1
week 2	October	20	week 9	December	15
week 3	October	27	week 10	January	12
week 4	November	3	week 11	January	19
week 5	November	10	week 12	January	26
week 6	November	17	1st exam	February	2
week 7	November	24			

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Topics

abstract data types, algebraic data types, binary search trees, combinator parsing, efficiency, encoding data types as lambda-terms, evaluation strategies, formal verification, first steps, guarded recursion, Haskell introduction, higher-order functions, historical overview, implementing a type checker, induction, infinite data structures, input and output, lambda-calculus, lazy evaluation, list comprehensions, lists, modules, pattern matching, polymorphism, property-based testing, reasoning about functional programs, recursive functions, sets, strings, tail recursion, trees, tupling, type checking, type inference, types, types and type classes, unification, user-defined types

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Overview

- History
- Notions
- A Taste of Haskell
- First Steps

History



1936 Alonzo Church: λ-calculus



1936 Alonzo Church: λ -calculus

1917 2017

1937 Alan Turing:







Alonzo Church: λ-calculus

1924 Moses Schönfinkel: combinatory logic

1917

ring:

2017

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Robin Milner: LCF. Standard

Miranda

MI



2003

Martin Odersky: Scala

2017

Haskell2010

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Notions

- variables point to storage locations in memory
- state is content of variables in scope at given execution point

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Example – Assignment

after x := 10, the location x has content 10 (the state changed)

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Side Effects

a function or expression has side effects if it modifies state

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Side Effects

a function or expression has side effects if it modifies state

Example $-\sum_{i=0}^n i$

```
count := 0
total := 0
while count < n
  count := count + 1
total := total + count</pre>
```

Example $-\sum_{i=0}^{n} i$

the Haskell way of summing up the numbers from 0 to n is sum [0..n]

- [0..4] generates list [0,1,2,3,4]
- sum is predefined function, summing up elements of a list

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Example – Defining Functions

• [m..n] computes range of numbers from m to n
range m n =
 if m > n then []
 else m : range (m + 1) n

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sum xs computes sum of elements in xs
mySum [] = 0
mySum (x:xs) = x + mySum xs

Pure Functions

a function is pure if it always returns same result on same input

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Counterexample – Random Numbers

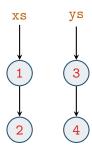
```
the C function rand (producing random numbers) is not pure
rand() = 0
rand() = 10
rand() = 42
```

data that does not change after initial creation

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Example – Linked Lists

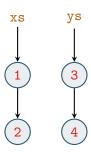
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data that does not change after initial creation

Example – Linked Lists

- consider two linked lists xs = [1,2] and ys = [3,4]
- after concatenation zs = xs ++ ys

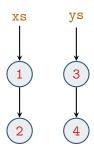


data that does not change after initial creation

Example – Linked Lists

append elements of ys to xs

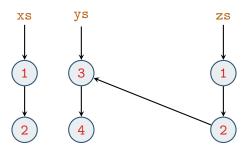
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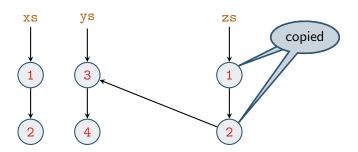
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Recursion

a function is recursive if it is used in its own definition

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Example – Factorial Numbers

```
factorial n =
  if n < 2 then 1
  else n * factorial (n - 1)</pre>
```

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- general idea: "replace equals by equals"

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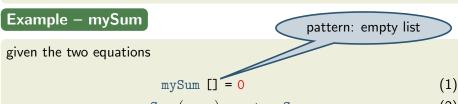
Example - mySum

given the two equations

$$mySum [] = 0$$
 (1)

$$mySum(x:xs) = x + mySum xs$$
 (2)

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Example – mySum

given the two equations pattern: list with "head" x and "tail" xs

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```
mySum [1,2,3]
```

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$$mySum [] = 0 (1)$$

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mySum [1,2,3] = 1 + mySum [2,3] using (2)
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given the two equations

$$mySum [] = 0$$

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mySum [1,2,3] = 1 + mySum [2,3] using (2)
= 1 + (2 + mySum [3]) using (2)
```

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A Taste of Haskell

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Example – Quicksort

- sort list of elements smaller than or equal to x
- sort list of elements larger than x
- insert x in between

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Example – Quicksort

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- sort list of elements larger than x
- insert x in between

```
qsort [] = []
qsort (x:xs) = qsort le ++ [x] ++ qsort gt
    where
    le = [a | a <- xs, a <= x] -- list comprehension
    gt = [b | b <- xs, b > x]
```

First Steps

Haskell on the Web

- main entry point www.haskell.org
- most widely used Haskell compiler: GHC
- with interpreter GHCi

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Starting the Interpreter (GHCi)

```
$ ghci
GHCi, version 8.0.2: http://www.haskell.org/ghc/
:? for help
...
Prelude>
```

The Standard Prelude

on startup GHCi loads the "Prelude," importing many standard functions

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Examples

```
• arithmetic: +, -, *, /, ^, mod, div
```

lists

```
drop n xs drop first n elements from list xs
head xs extract first element from list xs
length xs number of elements in list xs
product xs multiply elements of list xs
reverse xs as the name says: reverse list xs
sum xs sum up elements of list xs
tail xs obtain list xs without its first element
take n xs take first n elements from list xs
```

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```

 note: in code examples Prelude functions are denoted in green and others in blue; variables are denoted in dark orange

Function Application

- in mathematics: function application is denoted by enclosing arguments in parentheses, whereas multiplication of two arguments is often implicit (by juxtaposition)
- in Haskell: reflecting its primary status, function application is denoted silently (by juxtaposition), whereas multiplication is denoted explicitly by *

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Examples

Mathematics	Haskell
f(x)	f x
f(x,y)	f x y
f(g(x))	f (g x)
f(x,g(y))	f x (g y)
f(x) g(y)	f x * g y
f(a,b) + c d	f a b + c * d

Haskell Scripts

- define new functions inside scripts
- text file containing definitions
- common suffix .hs

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My First Script - test.hs

- set editor from inside GHCi :set editor vim
- start editor :edit test.hs and type
 double x = x + x
 quadruple x = double (double x)
- load script

```
Prelude> :load test.hs
[1 of 1] Compiling Main ( test.hs, interpreted )
Ok, modules loaded: Main.
```

*Main>

Interpreter Commands

```
Command
                  Meaning
:load (name) load script (name)
:reload
                reload current script
:edit (name)
                  edit script (name)
                  edit current script
:edit
                  show type of \langle expr \rangle
:type \(\langle expr\rangle\)
:set \langle prop \rangle
                  change various settings
:show (info)
                  show various information
:! ⟨cmd⟩
                  execute \langle cmd \rangle in shell
:?
                  show help text
                  bye-bye!
:quit
```

Example Session

```
> :load test.hs
> quadruple 10
40
> take (double 2) [1,2,3,4,5,6]
[1,2,3,4]
> :edit test.hs
factorial n = product [1..n]
average ns = sum ns `div` length ns
> :reload
> factorial 10
3628800
> average [1,2,3,4,5]
3
```

Example Session

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factorial n = product [1..n]
average ns = sum ns `div` length ns
> :reload
> factorial 10
3628800
> average [1,2,3,4,5]
3
                      enclosing function in `...` turns it infix
```

names of functions and their arguments have to conform to following syntax

```
\begin{array}{lll} \langle \textit{lower} \rangle & \stackrel{\text{def}}{=} & \text{a} \mid \dots \mid \text{z} \\ \langle \textit{upper} \rangle & \stackrel{\text{def}}{=} & \text{A} \mid \dots \mid \text{Z} \\ \langle \textit{digit} \rangle & \stackrel{\text{def}}{=} & \text{0} \mid \dots \mid \text{9} \\ \langle \textit{name} \rangle & \stackrel{\text{def}}{=} & (\langle \textit{lower} \rangle \mid \_)(\langle \textit{lower} \rangle \mid \langle \textit{upper} \rangle \mid \langle \textit{digit} \rangle \mid ", \mid \_)^* \end{array}
```

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Reserved Names

case class data default deriving do else foreign if import in infix infixr instance let module newtype of then type where $_$

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Examples

myFun fun1 arg_2 x'

The Layout Rule

- items that start in same column are grouped together
- by increasing indentation, items may span multiple lines
- groups end at EOF or when indentation decreases
- script content is group, start nested group by where, let, do, or of
- ignore layout: enclose groups in '{' and '}' and separate items by ';'

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Examples

```
main =
  let x = 1
    y = 1
  in
  putStrLn (take
    (x+y) (zs++us))
  where
    zs = []
    us = "abc"
```

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Examples

Comments

there are two kinds of comments

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- multi-line comments: enclosed in {- and -}

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Examples

```
-- Factorial of a positive number:

factorial n = product [1..n]

-- Average of a list of numbers:

average ns = sum ns `div` length ns

{- currently not used

double x = x + x

quadruple x = double (double x)

-}
```

Exercises (for October 20th)

- Read http://haskell.org/haskellwiki/Functional_programming and http://haskell.org/haskellwiki/Haskell_in_5_steps.
- 2. Work through lessons 1 to 3 on http://tryhaskell.org/.
- 3. Explain and correct the 3 syntactic errors in the script:
 N = a 'div' length xs

```
where
a = 10
xs = [1,2,3,4,5]
```

- 4. Show how the library function last (selecting the last element of a non-empty list) could be defined in terms of the Prelude functions used in this lecture. Can you think of another possible definition?
- 5. Show two possible definitions of the library function init (removing the last element from a list) in terms of the functions introduced so far.
- 6. Use recursion to define a function gcd, computing the greatest common divisor of two given numbers.

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