Functional Programming

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Lecture 3

Topics

abstract data types, algebraic data types, binary search trees, combinator parsing, efficiency, encoding data types as lambda-terms, evaluation strategies, formal verification, first steps, guarded recursion, Haskell introduction, higher-order functions, historical overview, implementing a type checker, induction, infinite data structures, input and output, lambda-calculus, lazy evaluation, list comprehensions, lists, modules, pattern matching, polymorphism, property-based testing, reasoning about functional programs, recursive functions, sets, strings, tail recursion, trees, tupling, type checking, type inference, types, types and type classes, unification, user-defined types

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Overview

- Module Basics
- Lists and Strings
- Recursive Functions
- Example – Printing a Calendar

Module Basics

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module Module (...) where
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Example

```
module Stack where
type Stack a = [a]
empty = []
push  = (:)
pop s = (head s, tail s)
```

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Example

```
push :: a -> Stack a -> Stack a
push = (:)
```

- note the partial application of `(:)`
- this is equivalent to `push x s = x : s`

Lists and Strings

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- the type `String` is just a type synonym for `[Char]`
- that is, strings are just lists of characters
- consequently, all list functions apply also to `String`s

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- `[]` is the same as `""` for strings
- `['h','e','l','l','o']` is the same as `"hello"` for strings

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Useful Functions on Strings

- `lines :: String -> [String]` – breaks string at newlines
- `unlines :: [String] -> String` – concatenates strings, inserting newlines
- `words :: String -> [String]` – breaks string at white space
- `unwords :: [String] -> String` – concatenates strings, separated by spaces

Interlude – Function Composition

- in mathematics $f \circ g$ usually denotes applying f after g
- more precisely, $(f \circ g)(x) = f(g(x))$
- only possible if output of g is compatible with input of f, that is, $f\colon B \to C$ and $g\colon A \to B$
- in Haskell: `(.) :: (b -> c) -> (a -> b) -> (a -> c)`
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Examples

- `map (f . g) xs` – on every element of `xs`, first apply `g` and then `f`
- equivalent to `map f (map g xs)`
- what are the results of `unwords . words` and `words . unwords`?

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Examples

- `length xs = sum [1 | _ <- xs]`
- `firsts ps = [x | (x, _) <- ps]`
- `flatMap f xs = [y | x <- xs, y <- f x]`

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Examples

- `[x | x <- [1..10], even x]`
- `find k t = [v | (k', v) <- t, k == k']`
- ``factors n = [x | x <- [1..n], n `mod` x == 0]``
- `primes = [n | n <- [1..], factors n == [1,n]]`

Recursive Functions

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  3. define simple cases (e.g., `product [] = 1`)
  4. define other cases (e.g., `product (x:xs) = x * product xs`)
  5. generalize and simplify (e.g., `product :: Num a => [a] -> a` and `product = foldr (*) 1`)

Example – drop

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- generalize and simplify:
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drop :: Integer -> [a] -> [a]
drop n xs | n <= 0  =  xs
drop _ []           =  []
drop n (_:xs)       =  drop (n - 1) xs
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- generalize and simplify
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Example – Printing a Calendar

Printing a Calendar

- given a month and a year, print the corresponding calendar
- separate construction phase (computation of days, leap year, … in file `Calendar.hs`) from printing
- we concentrate on printing, assuming machinery for construction

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Example – October 2017

```
Su Mo Tu We Th Fr Sa
 1  2  3  4  5  6  7
 8  9 10 11 12 13 14
15 16 17 18 19 20 21
22 23 24 25 26 27 28
29 30 31
```

The Picture Analogon

pictures:

- atomic part: pixel
- height and width
- white pixel

strings:

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Auxiliary Types

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type Height  = Int
type Width   = Int
type Picture = (Height, Width, [[Char]])
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- consider (`h`, `w`, `rs`)

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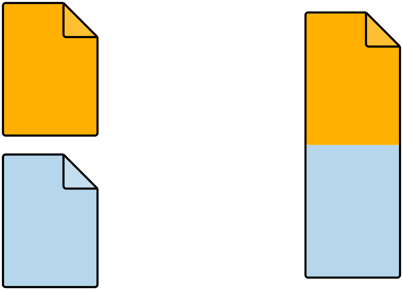
- consider (`h`, `w`, `rs`)
- `rs :: [[Char]]` – "list of rows"
- invariant 1: length of `rs` is height `h`
- invariant 2: all rows (that is, lists in `rs`) have length `w`

Stacking 2 Pictures Above Each Other



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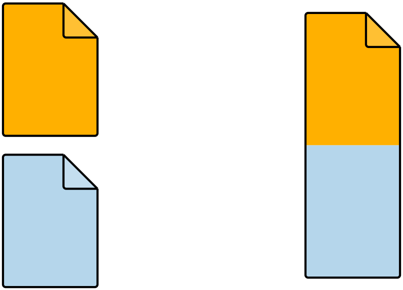

above

```
above :: Picture -> Picture -> Picture
(h, w, css) `above` (h', w', css')
  | w == w'    = (h + h', w, css ++ css')
  | otherwise = error "above: different widths"
```

Stacking Several Pictures Above Each Other

```
stack :: [Picture] -> Picture
stack = foldr1 above
```

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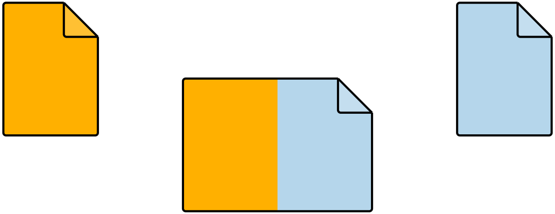
Notes

- `error :: String -> a`, indicates a runtime error, given as string
- `foldr1` – special version of `foldr`, without base value (does not work on empty lists)

  ```
  foldr1 :: (a -> a -> a) -> [a] -> a
  foldr1 f [x]     = x
  foldr1 f (x:xs) = x `f` foldr1 f xs
  ```

Spreading 2 Pictures Beside Each Other



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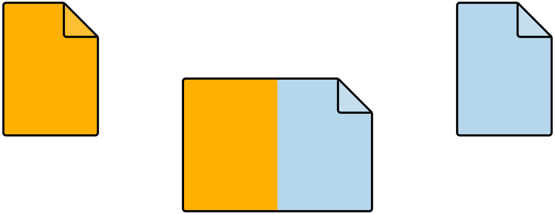

beside

```
beside :: Picture -> Picture -> Picture
(h, w, css) `beside` (h', w', css')
  | h == h'    = (h, w + w', zipWith (++) css css')
  | otherwise = error "beside: different heights"
```

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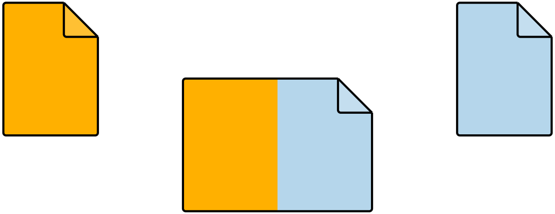

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Spreading Several Pictures Beside Each Other

```
spread :: [Picture] -> Picture
spread = foldr1 beside
```

Combining 2 Lists via a Function

- `zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]`
- `zipWith f` $[x_1, \ldots, x_m]$ $[y_1, \ldots, y_n] =$ $[x_1$ `` `f` `` $y_1, \ldots, x_{\min\{m,n\}}$ `` `f` `` $y_{\min\{m,n\}}]$
- specialization `zip :: [a] -> [b] -> [(a, b)]`

$$\texttt{zip} = \texttt{zipWith (,)}$$

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  `zip` = `zipWith (,)`

Examples

- `zip [1,2,3] ['a','b']` = `[(1,'a'),(2,'b')]`
- `zipWith (*) [1,2] [3,4,5]` = `[1*3,2*4]` = `[3,8]`
- `zipWith drop [1,0] ["a","b"]` =
  `[drop 1 "a",drop 0 "b"]` = `["","b"]`

Creating Pictures

- single pixels

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- blank

```
blank :: Height -> Width -> Picture
blank h w = (h, w, blanks)
  where
    blanks = replicate h (replicate w ' ')
```

Constructing a Month

- assume function monthInfo :: Int -> Int -> (Int, Int), returning the first weekday of the month together with the number of days for the month
- where days are 0 (Sunday), 1 (Monday), ...
- e.g., monthInfo 10 2017 = (0, 31), meaning that the first weekday of October 2017 is a Sunday and the month has 31 days

```haskell
daysOfMonth :: Int -> Int -> [Picture]
daysOfMonth m y =
  map (row . rjustify 3 . pic) [1 - d .. 42 - d]
  where
    (d, t) = monthInfo m y
    pic n = if 1 <= n && n <= t then show n else ""


month :: Int -> Int -> Picture
month m y = tile $ group 7 $ daysOfMonth m y
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variant of function application with lowest priority to avoid parentheses

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- `rjustify` – right-justify given text inside box of given width

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rjustify :: Int -> String -> String
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- `tile` – tile a list of lists of pictures

```haskell
tile :: [[Picture]] -> Picture
tile = stack . map spread
```

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- transform a `Picture` into a `String`

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  where
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- putting it all together in `Cal.hs`:

```
module Main where
import System.Environment -- for getArgs
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- compile: `ghc --make Cal`   run: `./Cal 10 2017`

Exercise Preparation – Caesar Cipher

- a Caesar cipher encodes text by replacing each letter by another one, some fixed positions (the key) down the alphabet
- for example, encoding `hello` with a key of 2, yields `jgnnq`
- in the following we restrict to lowercase letters
- approximate letter frequency list for English

```
tableEn = [8.2,1.5,2.8,4.3,12.7,2.2,2.0,6.1,7.0,
           0.2,0.8,4.0,2.4,6.7,7.5,1.9,0.1,6.0,
           6.3,9.1,2.8,1.0,2.4,0.2,2.0,0.1]
```

- chi-square statistic

$$\sum_{i=0}^{n-1} \frac{(os_i - es_i)^2}{es_i}$$

- where os is list of observed frequencies
- and es list of expected frequencies (e.g., `tableEn` for English)
- the lower chi-square, the better the match between os and es

Exercises (for November 3rd)

1. Read Chapter 3 of Real World Haskell.
2. Implement a function `rotate :: Int -> [a] -> [a]` that rotates the elements of a list to the left (wrapping around at the start of the list). For example, `rotate 3 [1,2,3,4,5]` = `[4,5,1,2,3]`.
3. Implement a function `encode :: Int -> String -> String` that applies a Caesar cipher, e.g., `encode 10 "hello world"` = `"rovvy gybvn"`. (Note that decoding is just encoding with the negated key.)
4. Implement a function `freqs :: String -> [Double]` that produces a frequency list for the 26 lowercase letters. For example, `freqs "aaab"` = `[75.0,25.0,0.0,...,0.0]`.
5. Implement the chi-square statistic by a function `chisqr :: [Double] -> [Double] -> Double`, taking two frequency lists.
6. Implement a function `crack :: String -> String` that is able to break the ciphertext `"zosk lux xkgr ktixevzout"`. You may use all the previous functions and `tableEn`.

Hints

- function `f` from module `M` is available through `M.f`
- in order to use `f` without qualifier `import M` at start of file
- converting between integers and characters:
  - `Data.Char.chr :: Int -> Char`
  - `Data.Char.ord :: Char -> Int`
- converting from integer to float: `fromIntegral`