Computational Logic

Functional Programming

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Lecture 12

Topics

abstract data types, algebraic data types, binary search trees, combinator parsing, efficiency, encoding data types as lambda-terms, evaluation strategies, formal verification, first steps, guarded recursion, Haskell introduction, higher-order functions, historical overview, implementing a type checker, induction, infinite data structures, input and output, lambda-calculus, lazy evaluation, list comprehensions, lists, modules, pattern matching, polymorphism, property-based testing, reasoning about functional programs, recursive functions, sets, strings, tail recursion, trees, tupling, type checking, type inference, types, types and type classes, unification, user-defined types

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Overview

- Lazyness and Infinite Data Structures

- Examples of (Infinite) Lazyness

Lazyness and Infinite Data Structures

Lazyness - Motivation

- only compute values needed for final result
- avoid computing same value twice (memoization)

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main = do
  input <- getLine
  let i = read input
  print (head [f1 i, f2 i])
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- value of `f2 i` not needed
- however, without lazyness program would not terminate

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- in Haskell (only using Prelude functions!)

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snd . head . filter (p . fst) $ zip [m..n] [0..]
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```

- (lazy) evaluation

```
snd . head . filter ((==1) . fst) $ zip [1..n] [0..]
  = ... $ (1,0) : zip [2..n] [1..]
  = snd . head $ (1,0) : filter ((==1) . fst) (...)
  = snd (1,0)
  = 0
```

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- without lazyness (ignoring nontermination of `[0..]`) three list traversals

Watch out for Memory Leaks

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Function (Tail Recursive)

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length' acc []       =  acc
length' acc (_:xs)   =  length' (acc + 1) xs
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Function (Tail Recursive)

```
length' acc []       =  acc
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```

Evaluation

```
length' 0 [1,2,3,4]
  =  length' (0+1) [2,3,4]
  =  length' (0+1+1) [3,4]
  =  length' (0+1+1+1) [4]
  =  length' (0+1+1+1+1) []
  =  (0+1+1+1+1)
```

Being Strict – The seq Function

- type `seq :: a -> b -> b`
- `x `seq` y` reduces `x` to WHNF and returns `y`

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Strict Folding of Lists – Data.List.foldl'

```
foldl' _ b []     = b
foldl' f b (x:xs) = c `seq` foldl' f c xs
  where
    c = f b x in
```

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foldl' f b (x:xs) = c `seq` foldl' f c xs
  where
    c = f b x in
```

Example

```
length'' = foldl' (const . (+1))
```

Examples of (Infinite) Lazyness

Example 1 – Fibonacci Numbers (this time starting from 0)

$$\mathsf{fib}(i) = \begin{cases} 0 & \text{if } i \leq 0 \\ 1 & \text{if } i = 1 \\ \mathsf{fib}(i-1) + \mathsf{fib}(i-2) & \text{otherwise} \end{cases}$$

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Sequence

$0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 233\ 377\ 610\ 987\ 1597\ 2584\ \ldots$

One Way of Computing Fibonacci Numbers

starting at 0 | 0 1

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1
$(+)$	

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1
$(+)$	

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1
(+)	1

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1
starting at 1	1	1	
$(+)$	1		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1
starting at 1	1	1	
$(+)$	1		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1
starting at 1	1	1	
$(+)$	1	2	

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2
starting at 1	1	1	2	
$(+)$	1	2		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2
starting at 1	1	1	2	
$(+)$	1	2		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2
starting at 1	1	1	2	
$(+)$	1	2	3	

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3
starting at 1	1	1	2	3	
$(+)$	1	2	3		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3
starting at 1	1	1	2	3	
$(+)$	1	2	3		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3
starting at 1	1	1	2	3	
$(+)$	1	2	3	5	

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5
starting at 1	1	1	2	3	5	
$(+)$	1	2	3	5		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5
starting at 1	1	1	2	3	5	
$(+)$	1	2	3	5		

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5
starting at 1	1	1	2	3	5	
$(+)$	1	2	3	5	8	

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5	8
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$(+)$	1	2	3	5	8		

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starting at 0	0	1	1	2	3	5	8
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One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5	8	13
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One Way of Computing Fibonacci Numbers

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One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5	8	13
starting at 1	1	1	2	3	5	8	13	
$(+)$	1	2	3	5	8	13	21	

One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3	5	8	13	21
starting at 1	1	1	2	3	5	8	13	21	
$(+)$	1	2	3	5	8	13	21		

One Way of Computing Fibonacci Numbers

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Ingredients

- function to shift sequence to left

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starting at 1	1	1	2	3	5	8	13	21	...	
$(+)$	1	2	3	5	8	13	21	34	...	

Ingredients

- function to shift sequence to left
- function to add two sequences

Fibonacci Numbers in Haskell

```haskell
fibs :: [Integer]
fibs = 0 : 1 : zipWith (+) fibs (tail fibs)
```

Example 2 – The Sieve of Eratosthenes

start with list of all natural numbers (from 2 on)

1. mark first element x as prime
2. remove all multiples of x
3. go to Step 1

The Sieve in Haskell

```haskell
primes :: [Integer]
primes = sieve [2..]
  where
    sieve (x:xs) =
      x : sieve [y | y <- xs, y `mod` x /= 0]
```

Example 3 - Lazy Shenanigans

- task – replace all elements of list by its maximum in single traversal

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- in Haskell

```
replaceAllByMax xs = ys
  where
    (m, ys) = maxAndReplaceAll m xs
    maxAndReplaceAll c =
      foldr (\x (m, ys) -> (max x m, c : ys)) (0, [])
```

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Resulting Data Dependencies for [1,2,3]

input: [1 , 2 , 3]

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Resulting Data Dependencies for [1,2,3]

result: (m , ys)

input: [1 , 2 , 3]

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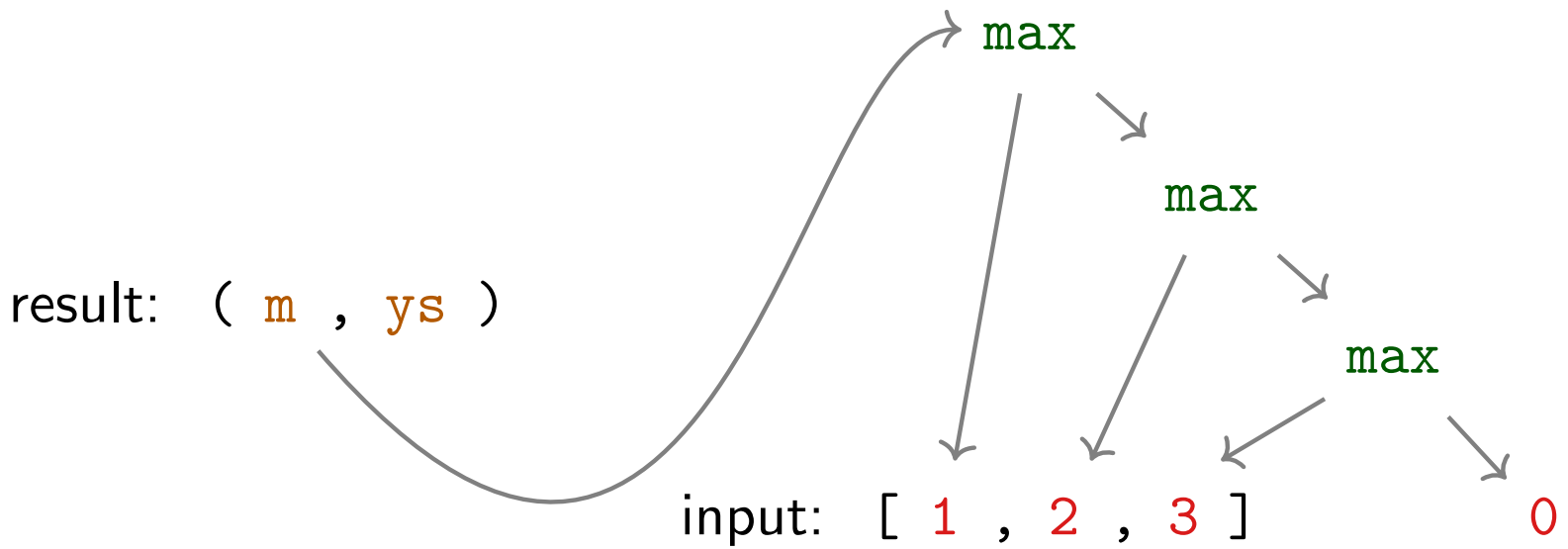

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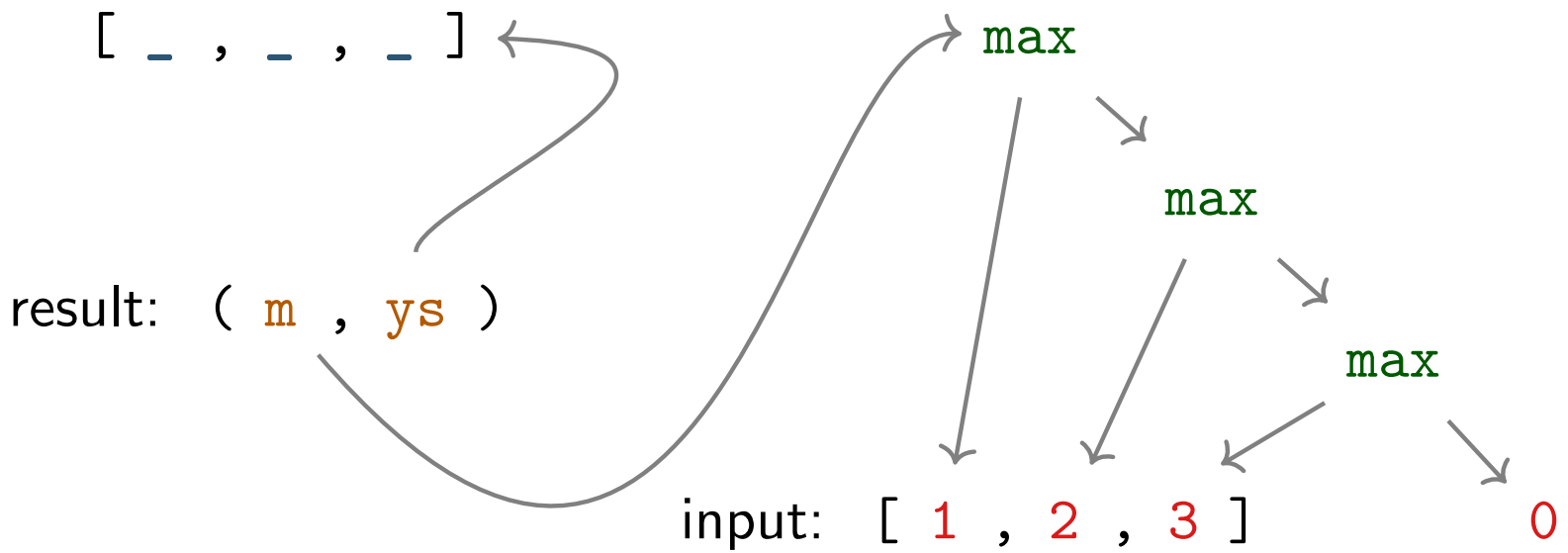

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