AC compatible simplification orders

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Main reference

References

[1] AKIHISA YAMADA, SARAH WINKLER, NAO HIROKAWA, and AART MIDDELDORP. AC-KBO revisited. *Theory and Practice of Logic Programming*, 16(2):163–188, 2016.

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  - TRS with AC-rules is not terminating

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  - $f(x, y) = f(y, x)$
  - $f(x, f(y, z)) = f(f(x, y), z)$

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- aim
  - termination modulo AC
  - accomplished by AC-compatible simplification order

Reminder

let R be a strict order

Rewrite relation

1. closed under contexts
   $s \, R \, t \to C[s] \, R \, C[t]$ for all contexts C

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Subterm property

R has subterm property if $C[s]\,R\,s$ for all non-empty contexts C and terms s

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Simplification order

simplification order is rewrite relation with subterm property

Rewriting modulo equations

ARS R terminates modulo E, set of equations, if no term t_1 with infinite chain

$t_1 =_E \cdot \rightarrow_R \cdot =_E t_2 =_E \cdot \rightarrow_R \cdot =_E t_3 \ldots$

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consider AC equations for all $f \in F_{AC}$

- $f(x, y) = f(y, x)$
- $f(x, f(y, z)) = f(f(x, y), z)$
- $f(f(x, y), z) = f(x, f(y, z))$

denote $=_E$ as $=_{AC}$

example:

- $1 + 2 \rightarrow 2 + 1$ is terminating
- $1 + 2 \rightarrow 2 + 1 =_{AC} 1 + 2 \rightarrow 2 + 1 \ldots$ not terminating modulo AC

AC-compatible simplification order

$>$ is AC-compatible simplification order if

- $>$ is simplification order
- $=_{AC} \cdot > \cdot =_{AC} \subseteq >$

example: $1 + 2 \to 2 + 1$

- assume $R \subseteq >$
- assume $>$ AC-compatible simplification order
- $\implies 1 + 2 \to 2 + 1 =_{AC} 1 + 2 \to 2 + 1 \ldots$
- $\implies 1 + 2 > 1 + 2$, $>$ not simplification order

Weight function

weight function (w, w_0) over signature F is defined

- $w_0 > 0$
- constant $c \in F \implies w(c) \geq w_0$

$$w(t) := \left\{ \begin{array}{ll} w_0, & \text{if } t \in V \\ w(f) + \sum_{i=1}^{n} w(t_i) & \text{if } t = f(t_1, \ldots, t_n) \end{array} \right\}$$

precedence $>$ over F then (w, w_0) is admissible if

- f unary
- $w(f) = 0$
- $f \neq g$

$\implies f > g$

KBO

- precedence $>$
- admissible weight function (w, w_0)

$s >_{KBO} t$

1. $|s|_x \geq |t|_x$ for all $x \in V$
2. $w(s) > w(t)$ or
3. $w(s) = w(t)$ and one of the following holds
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   2. $s = f(s_1, \ldots, s_m)$, $t = g(t_1, \ldots, t_n)$, $f > g$
   3. $s = f(s_1, \ldots, s_m)$, $t = f(t_1, \ldots, t_m)$, $(s_1, \ldots, s_m) >_{KBO}^{lex} (t_1, \ldots, t_m)$

Definitions

Top Flattening

$$\nabla_f(t) := \left\{ \begin{array}{ll} \{t\}, & \text{if } root(t) \neq f \\ \nabla_f(t_1) \uplus \nabla_f(t_2), & \text{if } t = f(t_1, t_2) \end{array} \right\}$$

example:

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- $\nabla_+(f(a) + g(b)) = \{f(a), g(b)\}$
- $\nabla_+(a + b + a) = \{a, a, b\}$

Definitions

extracting variables from term set denoted as

$$T\restriction_V := \{x \in T \mid x \in V\}$$

extracting terms where the root symbol is in relation to f

$$T\restriction_f^R := \{t \in T \setminus V \mid root(t)\, R\, f\}$$

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- $T = \{f(b), g(a), g(x), x, y\}$ then $T\restriction_V = \{x, y\}$
- precedence $f > + > g$ then $T\restriction_+^{\not<} := \{f(b)\}$

Definitions

$SR^f T$ denotes $S \upharpoonright_f^{\not<} R^{\text{mul}}\, T \upharpoonright_f^{\not<} \uplus T \upharpoonright_v - S \upharpoonright_v$

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$S >^+ T \Leftrightarrow \{f(a)\} >^{mul} \{f(b)\}$

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  4. $s = f(s_1, s_2)$, $t = f(t_1, t_2)$, $f \in F_{AC}$, $S = \nabla_f(s)$, $T = \nabla_f(t)$,
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    - $S >^{+} T \Leftrightarrow \{f(a)\} >^{mul} \{f(b)\}$

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  - $|S| > |T|$

Analyzed AC-KBO orders

paper analyzes three orders

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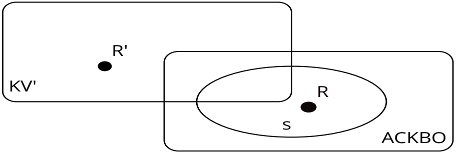
Relation to other orders



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- first rule implies $w(f) = 0$

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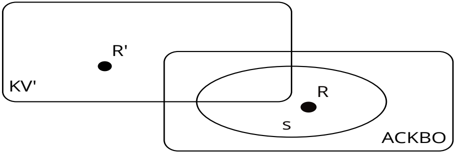

TRS R $\quad f(a+a) \to f(a) + f(a) \quad a + f(f(a)) \to f(a) + f(a)$

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- admissible implies $f > a$ and $f > +$

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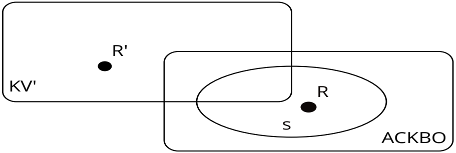

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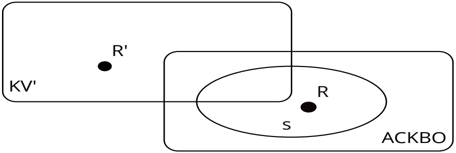

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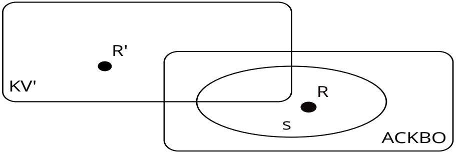

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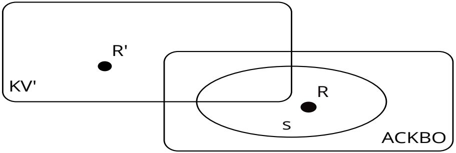

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- case $a > +$
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  - $f(f(a)) >_{ACKBO} f(a)$

Complexity

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does $s >_{AC-KBO} t$ hold

orientability problem:

- given TRS R

exists a weight function and precedence so that $R \subseteq >_{AC-KBO}$

Complexity

method	membership	orientability
Steinbach	P	?
ACKBO	P	NP-complete
KV	P	NP-complete
KV'	NP-complete	NP-complete
AC-RPO	NP-hard	NP-hard

Proof membership in polynomial time

lemma 1

- $\succsim \cdot \succ \cdot \succsim \subseteq \succ$
  - we call $(\succsim, \succ)$ an order pair
- $\sim := \succsim \setminus \succ$ symmetric
- if $s \sim t$ then $M \succ^{mul} N \Leftrightarrow M \uplus \{s\} \succ^{mul} N \uplus \{t\}$

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$M \succ^{mul} N$ if exist order $M = \{s_1 \ldots s_m\}$ and $N = \{t_1 \ldots t_n\}$ and exist $0 \leq k \leq min(m-1, n)$ where

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proof:
indices i, j with $s_i = s$ and $t_j = t$, k number of elements in the preorder case

- Case $i, j \leq k$

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proof:
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- Case $i, j \leq k$
- Case $i \leq k < j$

Proof membership in polynomial time

lemma 2

- $(\succsim, \succ)$ be an order pair
- $\sim := \succsim \setminus \succ$ is symmetric
- decision problem for $\succsim$ and $\succ$ are in P
- the decision problem for $\succ^{mul}$ is in P

proof multisets S and T

1. each $(s, t) \in S \times T$
   if $s \sim t$ then $S := S \setminus s$ and $T := T \setminus t$
2. each $t \in T$ search $s \in S$ such that $s \succ t$

membership in polynomial time follows from lemmas and induction argument

Proof sketch orientation

problem of infinitely many weight functions
solved by guessing the inequalities

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- substitute all variables to w_0
- TRS finitely many rules implies S finite

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- $S := \{t \mid s \in R_t \wedge s \trianglerighteq t\}$
- substitute all variables to w_0
- TRS finitely many rules implies S finite
- $3^{|S|-1}$ possible inequalities

Proof sketch orientation

problem of infinitely many weight functions
solved by guessing the inequalities

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- membership is in P
- solving set of inequalities is in P (Schrijver, 1986)
- set of inequalities has solution

Proof sketch orientation

problem of infinitely many weight functions
solved by guessing the inequalities

- TRS R, let $R_t := \{t \mid \exists u(tRu \lor uRt)\}$
- $S := \{t \mid s \in R_t \land s \trianglerighteq t\}$
- substitute all variables to w_0
- TRS finitely many rules implies S finite
- $3^{|S|-1}$ possible inequalities
- membership is in P
- solving set of inequalities is in P (Schrijver, 1986)
- set of inequalities has solution
- TRS is oriented

Further work

formalize ACKBO order in Isabelle/HOL:

reason for ACKBO order over the variants analyzed in paper

- includes Steinbach
- membership check in P

End

Thank you for your attention.

References

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