FORT 2.0

Franziska Rapp, supervised by Aart Middeldorp

University of Innsbruck
Computational Logic Group

January 10, 2018

Example (COPS 111)

TRS $\mathcal{R}$

$$\begin{array}{lll} a \to b & b \to d & d \to e \\ a \to c & c \to a & g(x) \to h(a) \\ a \to e & d \to a & h(x) \to e \end{array}$$

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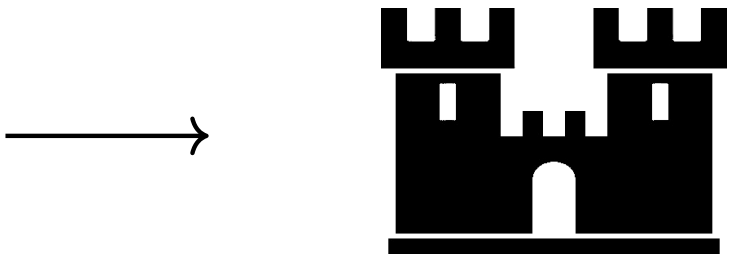
FORT

property  TRS

yes | no

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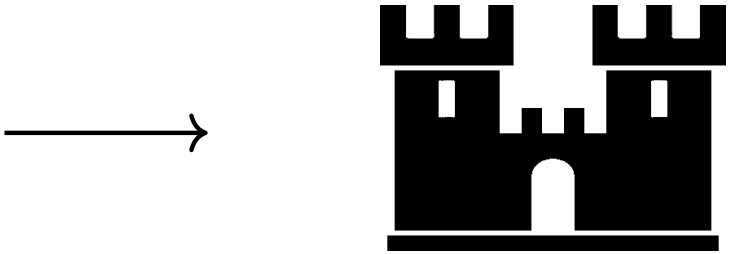
property  TRS

no | ?

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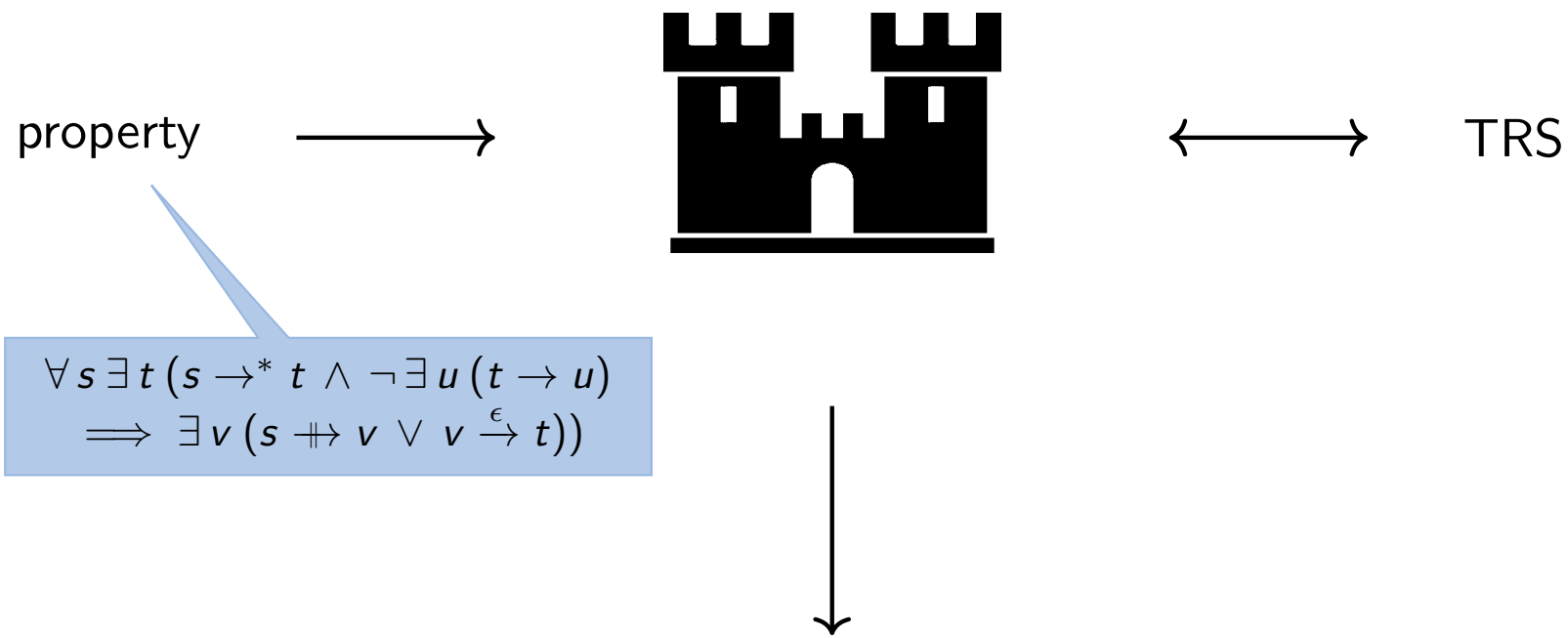
property
TRS
$\forall s \exists t (s \to^* t \wedge \neg \exists u (t \to u) \implies \exists v (s \twoheadrightarrow v \vee v \xrightarrow{\epsilon} t))$
yes | no | ?

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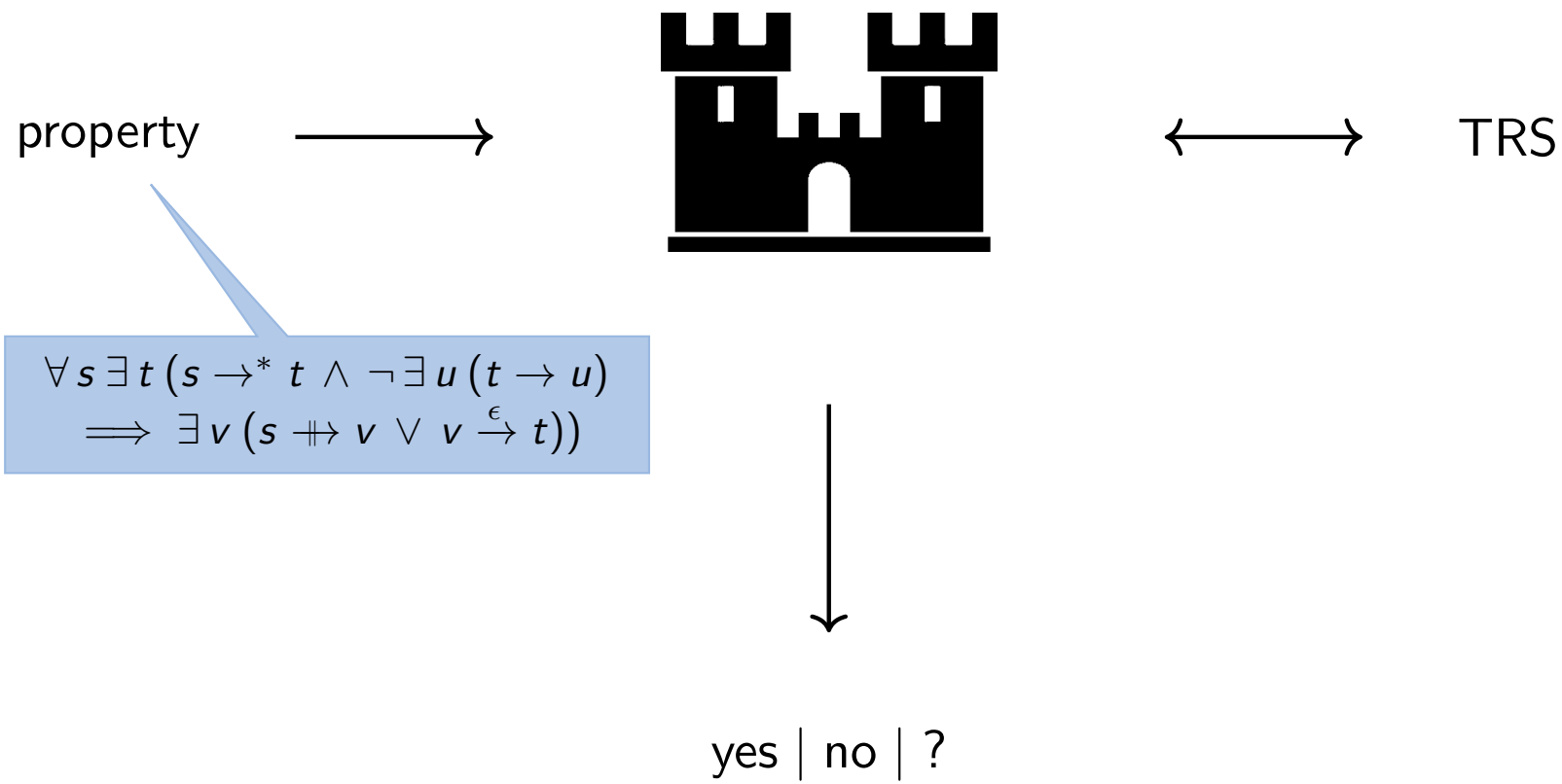

FORT is based on tree automata techniques (Dauchet and Tison, LICS 1990)

Outline

- First-Order Theory of Rewriting
- Automation
- Witness Generation
- Combinations of TRSs
- Extra Variables
- Summary

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- first-order logic over a language $\mathcal{L}$ without function symbols

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Derived Predicates

$$s \to^* t \iff s \to^+ t \vee s = t \qquad s \leftrightarrow t \iff s \to t \vee t \to s$$

$$s \to^! t \iff s \to^* t \wedge \neg \exists u\,(t \to u) \qquad s \downarrow t \iff \exists u\,(s \to^* u \wedge t \to^* u)$$

$$\mathrm{CR} \iff \forall s \forall t \forall u\,(s \to^* t \wedge s \to u \implies t \downarrow u)$$

SN can be expressed via additional finiteness predicate

Outline

Translation

- binary predicates are RR_2 relations and implemented via tree automata
  - ground tree transducers (GTTs) for $\not\Rightarrow$
  - RR_n automata for all n-ary relations

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- implications and universal quantifiers are eliminated

$$\varphi \Rightarrow \psi \;\equiv\; \neg\varphi \vee \psi \qquad\qquad \forall x\, \varphi \;\equiv\; \neg\exists x \neg\varphi$$

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Remark

formulas are not transformed into prenex normal form, since this increases the dimension of involved relations

Outline

Witness Generation for Formulas

- free variables (existentially quantified)

Example (1)

`-D -w COPS/39.trs "s ->! t & s ->! u & ~t = u"`

$f(b) \to a$ $\quad f(c) \to f(b)$ $\quad b \to e$ $\quad f(e) \to a$

$f(b) \to f(c)$ $\quad f(c) \to d$ $\quad c \to e'$ $\quad f(e') \to d$

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`Witnesses:` s : $f(b)$ $\quad t$: a $\quad u$: d

SAT

Witness Generation for Formulas

- free variables (existentially quantified)

Example (1)

```
-D -w COPS/39.trs "exists s, t, u (s ->! t & s ->! u & ~t = u)"
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Witness Generation for Formulas

- free variables (existentially quantified)
- universally quantified variables (in negative case)

Example (2)

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Witnesses for formula:  exists s, t, u (s -> t & s ->* u & ~t join u)
```

s : $f(c)$ $\quad$ t : $f(e')$ $\quad$ u : $f(e)$

NO

Function `find_terms`

Input:

- $\mathcal{A} = (\mathcal{F}', Q, Q_f, \Delta)$ RR_n automaton
- $\alpha \in Q$
- $Q_v \subseteq Q$

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`add` α `to` Q_v;

$\mathcal{C} = \{fs(\alpha_1, \ldots, \alpha_m) \to \alpha \in \Delta \mid \alpha_1, \ldots, \alpha_m \notin Q_v$, and if $Q_v = \{\alpha\}$ then $\bot \notin fs\}$;

avoids ending up in loops

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`while` $\mathcal{C} \neq \varnothing$ `do`

`select` $fs(\alpha_1, \ldots, \alpha_m) \to \alpha$ `from` $\mathcal{C}$ `with minimal` m;

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while $\mathcal{C} \neq \varnothing$ do

select $fs(\alpha_1, \ldots, \alpha_m) \to \alpha$ from $\mathcal{C}$ with minimal m;

for all $1 \leqslant i \leqslant m$ try $\overrightarrow{t_i} = \mathtt{find\_terms}(\mathcal{A}, \alpha_i, Q_v)$;

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inner calls might end up in loops

`od; fail`

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`while` $\mathcal{C} \neq \varnothing$ `do`

$fs = f_1 \cdots f_n$

`select` $fs(\alpha_1, \ldots, \alpha_m) \to \alpha$ `from` $\mathcal{C}$ `with minimal` m;
`for all` $1 \leqslant i \leqslant m$ `try` $\overrightarrow{t_i} =$ `find_terms`$(\mathcal{A}, \alpha_i, Q_v)$;
`return` $f_1(\pi_1 \overrightarrow{t_1}, \ldots, \pi_1 \overrightarrow{t_{a(f_1)}}), \ldots, f_n(\pi_n \overrightarrow{t_1}, \ldots, \pi_n \overrightarrow{t_{a(f_n)}})$
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`od; fail`

Witnesses for RR_n Automaton

Use `find_terms`$(\mathcal{A}, \alpha, \varnothing)$ with some $\alpha \in Q_f$ to generate witnesses for $\mathcal{A}$.

Example (Witness Generation)

Consider the RR_2 automaton $\mathcal{A}$ with final state ✓ and Δ:

$$\mathsf{aa} \to \alpha \qquad \bot\mathsf{a} \to \alpha' \qquad \mathsf{fg}(\alpha, \beta') \to \gamma \qquad \mathsf{fh}(\alpha, \beta, \alpha') \to ✓$$
$$\mathsf{bb} \to \beta \qquad \mathsf{b}\bot \to \beta' \qquad \mathsf{gf}(\beta, \alpha') \to \gamma \qquad \mathsf{ff}(\alpha, \gamma) \to ✓$$
$$\mathsf{gg}(\gamma) \to \gamma \qquad \mathsf{ff}(✓, \alpha) \to ✓$$

over the signature $\mathcal{F} = \{a : 0,\ b : 0,\ g : 1,\ f : 2,\ h : 3\}$.

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`find_terms`$(\mathcal{A}, ✓, \varnothing)$

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$\texttt{find\_terms}(\mathcal{A}, ✓, \varnothing)$

$Q_v = \{✓\}$

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Consider the RR_2 automaton $\mathcal{A}$ with final state $\checkmark$ and Δ:

$$\mathsf{aa} \to \alpha \qquad \bot\mathsf{a} \to \alpha' \qquad \mathsf{fg}(\alpha, \beta') \to \gamma \qquad \mathsf{fh}(\alpha, \beta, \alpha') \to \checkmark$$
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over the signature $\mathcal{F} = \{a : 0,\ b : 0,\ g : 1,\ f : 2,\ h : 3\}$.

$\texttt{find\_terms}(\mathcal{A}, \checkmark, \varnothing)$

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$\texttt{find\_terms}(\mathcal{A}, \alpha, \{\checkmark\})$

Example (Witness Generation)

Consider the RR_2 automaton $\mathcal{A}$ with final state ✓ and Δ:

$$\mathsf{aa} \to \alpha \qquad \bot\mathsf{a} \to \alpha' \qquad \mathsf{fg}(\alpha, \beta') \to \gamma \qquad \mathsf{fh}(\alpha, \beta, \alpha') \to ✓$$
$$\mathsf{bb} \to \beta \qquad \mathsf{b}\bot \to \beta' \qquad \mathsf{gf}(\beta, \alpha') \to \gamma \qquad \mathsf{ff}(\alpha, \gamma) \to ✓$$
$$\mathsf{gg}(\gamma) \to \gamma \qquad \mathsf{ff}(✓, \alpha) \to ✓$$

over the signature $\mathcal{F} = \{a : 0,\ b : 0,\ g : 1,\ f : 2,\ h : 3\}$.

$\texttt{find\_terms}(\mathcal{A}, ✓, \varnothing)$

$Q_v = \{✓\}$

$\mathcal{C} = \{\mathsf{fh}(\alpha, \beta, \alpha') \to ✓, \mathsf{ff}(\alpha, \gamma) \to ✓\}$

$\texttt{find\_terms}(\mathcal{A}, \alpha, \{✓\})$

$Q_v = \{✓, \alpha\}$

Example (Witness Generation)

Consider the RR_2 automaton $\mathcal{A}$ with final state $\checkmark$ and Δ:

$$\begin{array}{llll} \mathsf{aa} \to \alpha & \bot\mathsf{a} \to \alpha' & \mathsf{fg}(\alpha, \beta') \to \gamma & \mathsf{fh}(\alpha, \beta, \alpha') \to \checkmark \\ \mathsf{bb} \to \beta & \mathsf{b}\bot \to \beta' & \mathsf{gf}(\beta, \alpha') \to \gamma & \mathsf{ff}(\alpha, \gamma) \to \checkmark \\ & & \mathsf{gg}(\gamma) \to \gamma & \mathsf{ff}(\checkmark, \alpha) \to \checkmark \end{array}$$

over the signature $\mathcal{F} = \{a : 0,\ b : 0,\ g : 1,\ f : 2,\ h : 3\}$.

$\texttt{find\_terms}(\mathcal{A}, \checkmark, \varnothing)$

$Q_v = \{\checkmark\}$

$\mathcal{C} = \{\mathsf{fh}(\alpha, \beta, \alpha') \to \checkmark, \mathsf{ff}(\alpha, \gamma) \to \checkmark\}$

$\texttt{find\_terms}(\mathcal{A}, \alpha, \{\checkmark\})$

$Q_v = \{\checkmark, \alpha\}$

$\mathcal{C} = \{\mathsf{aa} \to \alpha\}$

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Consider the RR_2 automaton $\mathcal{A}$ with final state $\checkmark$ and Δ:

$$\begin{array}{llll} \mathsf{aa} \to \alpha & \bot\mathsf{a} \to \alpha' & \mathsf{fg}(\alpha, \beta') \to \gamma & \mathsf{fh}(\alpha, \beta, \alpha') \to \checkmark \\ \mathsf{bb} \to \beta & \mathsf{b}\bot \to \beta' & \mathsf{gf}(\beta, \alpha') \to \gamma & \mathsf{ff}(\alpha, \gamma) \to \checkmark \\ & & \mathsf{gg}(\gamma) \to \gamma & \mathsf{ff}(\checkmark, \alpha) \to \checkmark \end{array}$$

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$Q_v = \{\checkmark\}$

$\mathcal{C} = \{\mathsf{fh}(\alpha, \beta, \alpha') \to \checkmark, \mathsf{ff}(\alpha, \gamma) \to \checkmark\}$

$\overrightarrow{t_1} = (\mathsf{a}, \mathsf{a})$

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Outline

Properties Involving Multiple TRSs

- express properties on combinations of TRSs

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Remark

Search space for synthesis explodes.

Related Work

Carpa+ (Zantema, 2013) is tool for synthesizing TRSs that satisfy properties which can be encoded as SMT problems

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$$a \to b \qquad a \to f(b) \qquad f(a) \to b$$

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- properties are restricted to those that can be encoded into conjunctive fragment of SMT-LRA (linear real arithmetic)
- predecessor tool Carpa synthesized combinations of ARSs with SAT solver

Examples (1)

-  `-S -g "0 0 0" "[0]SN & [1]SN & ~SN & forall s, t, u ([0]s -> t & [1]t -> u => [0]s ->+ u | [1]s ->+ u)"`

Examples (1)

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input for Carpa:
3 (nr of elements)
2 (nr of basic relations)
x1=tc(1)
x2=tc(2)
irr(x1)
irr(x2)
x3=union(1,2)
x3=tc(x3)
nirr(x3)
x4=comp(1,2)
x5=union(x1,x2)
subs(x4,x5)

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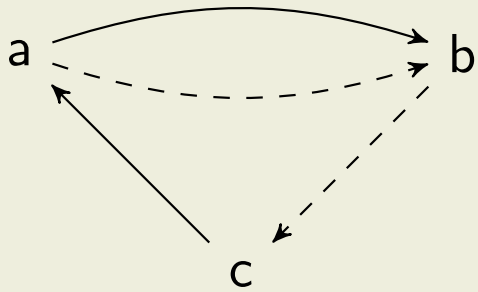
  generates TRSs



TRS 0
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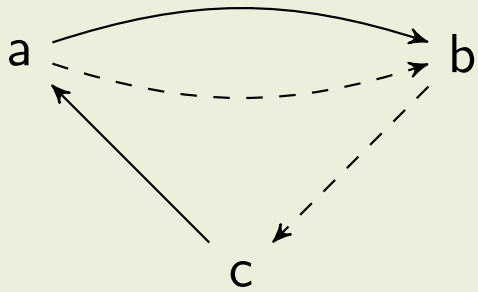

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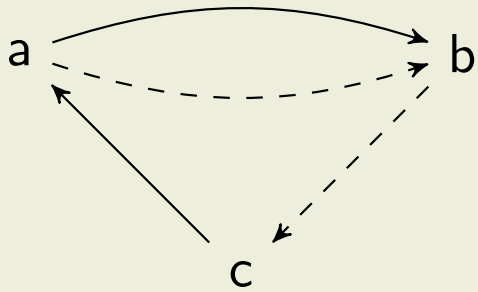

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```
LCom = forall s,t,u([0]s -> t & [1]s -> u => exists v([1]t ->* v & [0]u ->* v))
Com = forall s,t,u([0]s ->* t & [1]s ->* u => exists v([1]t ->* v & [0]u ->* v))
```

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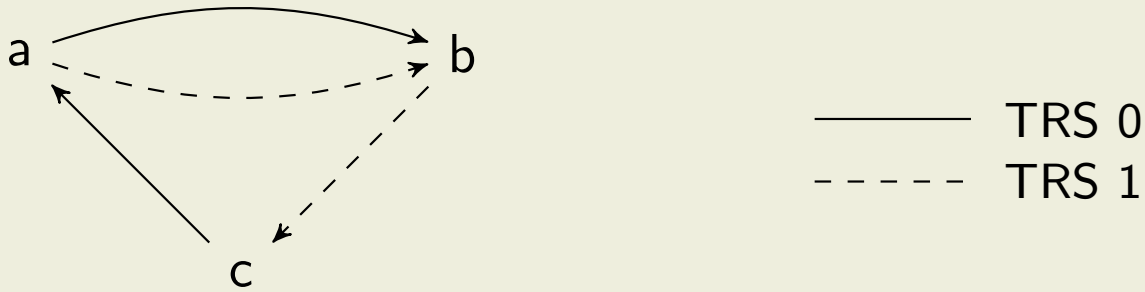
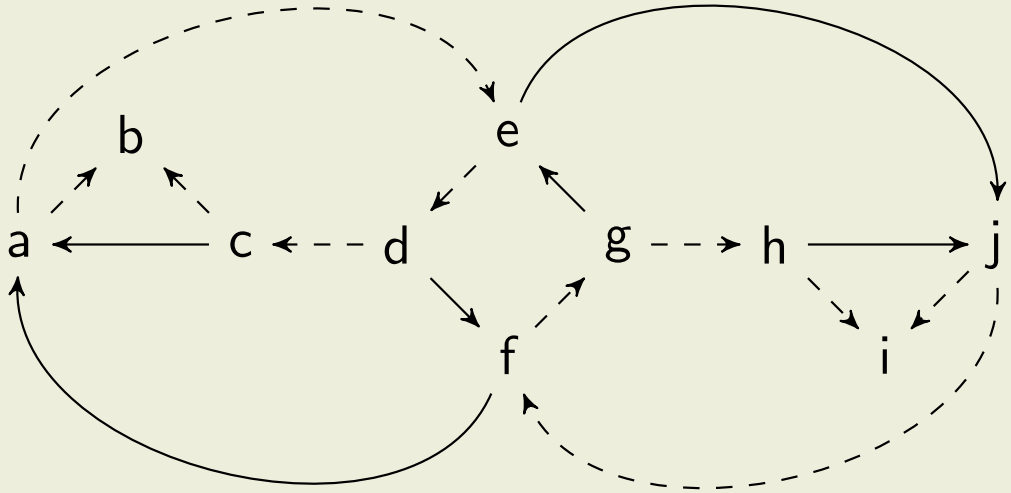


-  `-S -g "0 0 0" "[0]CR & [1]CR & LCom(0,1) & ~Com(0,1)"`

  generates TRSs

Examples (2)

```
-D -w Zantema/example8.trs "[0](CR & SN) & [1](CR & SN) & ~CR &
forall s, t, u ([0]s -> t & [1]s -> u =>
                exists v (([1]t -> v | [0]t -> v) & [0]u ->* v))"
```



Examples (2)

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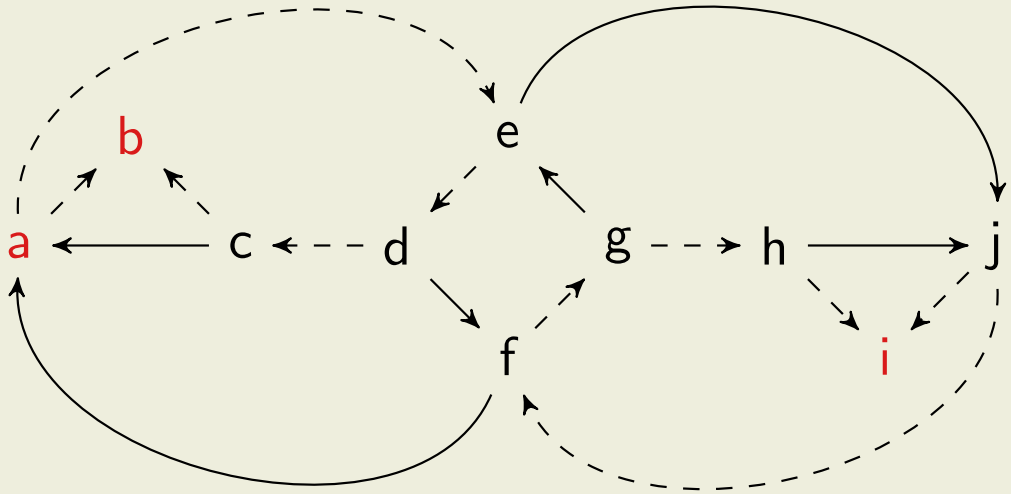
```




```
Witnesses for formula:  exists s, t, u (s -> t & s ->* u & ~t join u)
```

s : a t : b u : i

YES

Commutation

- Formula: `forall s,t,u ([0]s ->* t & [1]s ->* u =>`
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	FORT	CoLL
YES	613	451
NO	1379	81
MAYBE	153	1613

uniformly renamed TRSs

Outline

Extension

- variables on right hand sides, but rule has to be linear

Extension

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no variable should occur
more than once within a rule

Extension

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- $\mathsf{a} \to x \quad \mathsf{f}(x, y) \to \mathsf{g}(z)$

Extension

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- approximation mappings for call by need strategies

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Example (COPS 81)

$g(f(a)) \to f(g(f(a)))$ $\quad$ $g(f(a)) \to f(f(a))$ $\quad$ $f(f(a)) \to f(a)$

produced an RR_2 automaton of size 118 for $\leftrightarrow^*$ instead of size 427

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