Formal Languages and Automata Theory

This exam consists of four exercises. *Explain how you solved each exercise.* The available points for each item are written in the margin. You need at least 50 points to pass.

1. Consider the following three regular expressions over the alphabet $\Sigma = \{a, b\}$:

$$r_1 = (a^*b)^*a(ab)^* \qquad r_2 = (a^*ba)^*a(aa+b)^* \qquad r_3 = (ab)^*a(a^*b)^*$$

and the homomorphism φ defined by $\varphi(a) = ab$ and $\varphi(b) = \epsilon$.

[9] (a) Define for all $1 \leqslant i, j \leqslant 3$ with $i \neq j$ a string x_{ij} such that $x_{ij} \in L(r_i) - L(r_j)$.

[9] (b) Construct a DFA that accepts $L(r_1)$.

[8] (c) Compute $\varphi(L(r_2))$.

[8] (d) Compute $\varphi^{-1}(L(r_3))$.

2. Consider the pushdown automaton $M = (\{1,2\}, \{a,b\}, \{\perp, a, b\}, \delta, 1, \perp, \{2\})$ with δ consisting of the following transitions:

$$((1,\epsilon,\perp),(1,a\perp)) \qquad ((1,a,a),(1,ba)) \qquad ((1,b,b),(1,aa)) \qquad ((2,a,a),(2,\epsilon))$$
$$((1,a,a),(2,\epsilon)) \qquad ((1,b,b),(2,b)) \qquad ((2,\epsilon,\perp),(2,\epsilon))$$

[9] (a) Which of the following strings belong to $L_f(M)$, the language that M accepts by final state?

i. *ababa*

ii. *abaaa*

iii. *babab*

[9] (b) Does $L_f(M)$ coincide with $L_e(M)$, the language that M accepts by empty stack?

3. Determine whether the following subsets of $\{a,b,c,d\}^*$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free. (Here n, k, m, and l range over the natural numbers.)

[10] (a) $\{a^n b^m c^k d^l \mid n = 2k \text{ or } 2m = 3l\}$

[10] (b) $\{a^n b^m c^k d^l \mid n = 2k \text{ and } 2m = 3l\}$

[10] (c) $\{a^n b^m c^k d^l \mid n = 2l \text{ and } 2k = 3m\}$

[9] 4. (a) Let L_1 and L_2 be languages over the same alphabet Σ such that L_1 is recursive, $L_1 \cup L_2$ is recursive enumerable, and $L_1 \cap L_2 = \varnothing$. Prove that L_2 is recursive enumerable.

[9] (b) Show that the result of part (a) need not be true without the condition $L_1 \cap L_2 = \varnothing$.