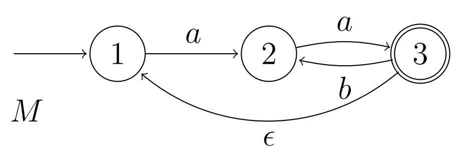
Formal Languages and Automata Theory SS 2010 LVA 703025

EXAM 1 July 1, 2010

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

$\boxed{1}$ Consider the regular expressions $\alpha = b^*((a+b)^* + c^*)ba$ and $\beta = a^*((ab)^* + bb)^*a$, the finite automaton



and the homomorphism h defined by $h(a) = cb$, $h(b) = \epsilon$, and $h(c) = bb$.

[4] (a) Determine whether the strings aaa, $baaba$, $abbba$, and $bbcaba$ belong to $L(\alpha)$ and $L(\beta)$.

[4] (b) Compute $h(L(\alpha))$.

[6] (c) Construct a finite automaton that accepts $L(\beta)$.

[6] (d) Construct a regular expression for $L(M)$ using one of the algorithms presented in the lecture.

$\boxed{2}$ Consider the NPDA $M = (\{q\}, \{a, b, c\}, \{\sharp, \bot\}, \delta, q, \bot, \varnothing)$ where δ consists of the following transitions:

$$((q, a, \bot), (q, \sharp\sharp\bot)) \qquad ((q, a, \sharp), (q, \sharp\sharp\sharp)) \qquad ((q, b, \sharp), (q, \epsilon)) \qquad ((q, \epsilon, \bot), (q, \epsilon))$$

[6] (a) Determine whether the strings ϵ, abb and abc belong to $L_e(M)$.

[5] (b) Construct an NPDA N such that $L_f(N) = L_e(M)$.

[5] (c) Construct a context-free grammar G such that $L(G) = L_e(M)$.

[2] (d) Is the grammar G constructed in (c) in Greibach normal form?

[2] (e) Is the grammar G constructed in (c) LL(1)?

$\boxed{3}$ Determine whether the following sets are (i) regular, (ii) context-free but not regular, or (iii) not context-free.

[7] (a) $A = \{x \in \{a, b, c\}^* \mid \#a(x) + \#b(x) - \#c(x) = 0\}$

[6] (b) $B = \{x \in \{a, b, c\}^* \mid \#c(x) = \min(\#a(x), \#b(x))\}$

[7] (c) $C = {\sim}\{xc(\mathbf{rev}\ x) \mid x \in \{a, b\}^*\}$

$\boxed{4}$ Consider the following decision problem P:

input:	TM M over the input alphabet $\{a, b\}$
question:	$L(M) \nsubseteq \{a\}^*$?

[7] (a) Prove that P is undecidable.

[7] (b) Is P semi-decidable? Prove your answer.

[6] (c) Consider the sets $A = \{M \mid L(M) = \varnothing\}$, $B = \{M \mid L(M)$ is not context-free$\}$, and $C = \{M \mid L(M)$ is infinite$\}$ of encodings of TMs. Complete the following table:

	recursive	r.e.
$\sim A$	$\square$	$\square$
B	$\square$	$\square$
$\sim C \cap B$	$\square$	$\square$

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
Context-free sets are closed under intersection.
The set $\{xx \mid x \in A\}$ is regular for every regular set A.
If $A \leq_m B$ and A is recursive then B is recursive.
Regular expressions without concatenation generate only finite sets.
The set $\{A \mid A \subseteq \{a\}^*\}$ is countably infinite.
Given an NFA $N = (Q, \Sigma, \Delta, S, F)$, the relation $\equiv_N$ defined by $x \equiv_N y$ if and only if $\widehat{\Delta}(S, x) \cap \widehat{\Delta}(S, y) \neq \varnothing$ is a Myhill-Nerode relation for $L(N)$.
Every context-free set is generated by a context-sensitive grammar.
If $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$.
The set $\{x \mid xx \in \{a, b\}^*\}$ is context-free.
Deterministic context-free sets are closed under homomorphic preimage.