Formal Languages and Automata Theory

This exam consists of four exercises. *Explain how you solved each exercise.* The available points for each item are written in the margin. You need at least 50 points to pass.

1. Consider the following three regular expressions over the alphabet $\Sigma = \{a, b\}$:

$$r_1 = ((a+b)^*a)^*b(a+b) \qquad r_2 = ba^*ba^*b \qquad r_3 = a^*ba^*ba^*$$

and the homomorphism φ defined by $\varphi(a) = b$ and $\varphi(b) = ba$.

[7] (a) Define for all $1 \leqslant i, j \leqslant 3$ with $i \neq j$ a string x_{ij} such that $x_{ij} \in L(r_i) - L(r_j)$.

[7] (b) Construct a DFA that accepts $L(r_1)$.

[7] (c) Compute $\varphi(L(r_2))$.

[7] (d) Compute $\varphi^{-1}(L(r_3))$.

2. Consider the pushdown automaton $M = (\{1,2\}, \{a,b\}, \{\perp, a, b\}, \delta, 1, \perp, \{2\})$ with δ consisting of the following transitions:

$$((1,\epsilon,\perp),(1,a\perp)) \quad ((1,a,a),(1,ba)) \quad ((1,b,b),(1,aa)) \quad ((2,a,a),(2,\epsilon))$$
$$((1,a,a),(2,\epsilon)) \quad ((1,b,b),(2,b)) \quad ((2,\epsilon,\perp),(2,\epsilon))$$

[8] (a) Which of the following strings belong to $L_f(M)$, the language that M accepts by final state?

i. *ababa*
ii. *abaaa*
iii. *babab*

[8] (b) Repeat the previous question for $L_e(M)$, the language that M accepts by empty stack.

[8] (c) Prove that $(ab)^i a^{2i+1} \in L_e(M)$ for every $i \geqslant 0$.

3. Determine whether the following sets are (i) regular, (ii) context-free but not regular, or (iii) not context-free.

[9] (a) $\sim\{a^ib^jc^k \mid i,j,k \leqslant 2 \text{ and } i \neq j+k\}$

[9] (b) $\{a^ib^jc^k \mid 0 < i = j < k\}^*$

[9] (c) $\{a^ib^jc^k \mid i,j,k \geqslant 0 \text{ and } jk > 3\}$

4. Consider the following decision problem P:

instance: TM M with input alphabet $\{a, b\}$
question: does M accept a palindrome (i.e., a string x such that $x = \mathbf{rev}\, x$)?

[7] (a) Show that P is undecidable.

[7] (b) Is P semi-decidable?

[7] (c) Is the complement of P semi-decidable?