Formal Language and Automata Theory WS 2012/2013 LVA 703603

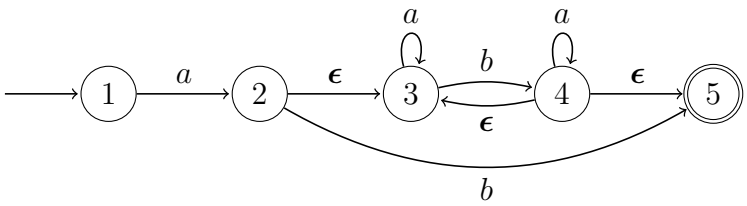
EXAM 1 January 28, 2013

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

1 Consider the three regular expressions

$$\alpha_1 = a((a^*b)^*a^* + b) \qquad \alpha_2 = a^*(ba^*)^* + ab \qquad \alpha_3 = a^*a(ba^*)^* + ab$$

and the NFA_ϵ N



[10] (a) For all $i, j \in \{1, 2, 3\}$ with $i \neq j$, either find a string showing that $L(\alpha_i) \neq L(\alpha_j)$ or prove that $L(\alpha_i) = L(\alpha_j)$.

[10] (b) Construct a DFA D such that $L(D) = L(N)$.

[10] 2 (a) Complete the transition function δ such that the NPDA

$$N = (\{1, 2\}, \{a, b\}, \{A, B, \bot\}, \delta, 1, \bot, \{2\})$$

accepts the set $A = \{x \mid \#a(x) > \#b(x)\}$ with final state. Here

$\delta(1, b, \bot) = (1, B\bot)$ $\quad \delta(1, b, B) =$ $\quad \delta(1, a, B) =$

$\delta(1, a, \bot) = (1, A\bot)$ $\quad \delta(1, a, A) =$ $\quad \quad =$

$\delta(1, \epsilon, A) =$

[10] (b) Consider the following context-free grammar G:

$$S \to SS \mid AU \mid AV \mid AB \mid a \qquad U \to SB \qquad V \to AV \mid a \qquad A \to a \qquad B \to b$$

Use the CKY algorithm to determine whether the strings

i. $aabba$
ii. $abbb$

belong to $L(G)$.

3 Determine whether the following sets over $\Sigma = \{0, 1\}$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free.

[10] (a) $A = \{0^i 1^j 0^k \mid 0 < i = j < k\}$

[10] (b) $B = \sim\{0^i 1^j 0^k \mid i, j, k \leqslant 3 \text{ and } i \neq j + k\}$

4 Consider the following decision problem P:

input: CFG G with terminal alphabet $\{a, b\}$
question: does G generate all strings except a?

[10] (a) Prove that P is undecidable.

[10] (b) Is the complement of P semi-decidable? Prove your answer.

[20] 5 Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement

The set VALCOMPS(M, x) is regular for every TM M and string x.

The regular expressions $(a^*(a^*b))^*(b^*a)^*$ and $(a + b + \boldsymbol{\epsilon})^*$ are equivalent.

The string aAa is a sentential form of the context-free grammar $(\{S, A\}, \{a\}, \{S \to SSA \mid AS \mid a, A \to a\}, S)$.

The relation $\equiv_A$ is a Myhill-Nerode relation for any context-free set A.

Emptiness is an undecidable property of context-free sets.

$(\boldsymbol{\varnothing} + \boldsymbol{\epsilon})(a + b^*)(a^* + b)\uparrow$

The set $\sim\{xx \mid x \in \{a, b\}^*\}$ is regular.

Deterministic context-free sets are closed under concatenation.

The membership problem for PDAs is decidable.

The context-sensitive grammar $(\{S\}, \{a, b\}, \{S \to ab, AB \to S\}, S)$ is in Kuroda normal form.