Formal Language and Automata Theory   WS 2014/2015   LVA 703603

EXAM 1   February 2, 2015

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

$\boxed{1}$ Consider the following two regular expressions:

$$\alpha = (ab)^*(bb)^*a^* \qquad\qquad \beta = (a^*b)^*b^*(ba + \boldsymbol{\epsilon})$$

[4] (a) Compute α_b and β_a.

[8] (b) Use derivatives to prove that $L(\alpha) \neq L(\beta)$.

[8] (c) Construct a DFA M such that $L(M) = {\sim} L(\alpha)$.

$\boxed{2}$ Consider the following context-free grammar G over $\Sigma = \{a, \$\}$:

$$S \to A\$B \mid B\$A \mid C \qquad\qquad B \to \$ \mid aBa$$
$$A \to \epsilon \mid aA \qquad\qquad C \to \$A\$ \mid aCa$$

[7] (a) Transform G into an equivalent grammar in Chomsky normal form.

[7] (b) Use the CKY algorithm to determine whether the string $a\$\$aa$ belongs to $L(G)$.

[6] (c) Construct an NPDA that accepts the set $\{a^n\$a^m\$a^n \mid n, m \geqslant 0\}$ by final state.

$\boxed{3}$ Determine whether the following sets over $\Sigma = \{a, b\}$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free.

[7] (a) $A = \{xax \mid x \in \{a\}^*\}$

[7] (b) $B = \{xax \mid x \in \Sigma^*\}$

[7] (c) $C = \{xa(\mathbf{rev}\ x) \mid x \in \Sigma^*\}$

$\boxed{4}$ Consider the following decision problems, where the terminal alphabet of the CFG and the input alphabet of the DFA are assumed to be the set $\{a, b\}$:

input:	CFG G, DFA M	input:	CFG G, DFA M
question:	$L(G) \subseteq L(M)$?	question:	$L(M) \subseteq L(G)$?

One of these problems is decidable and the other is not.

[9] (a) Which problem is decidable? Give a proof.

[10] (b) Prove that the other problem is undecidable. You may assume that the problem whether $L(G) = \{a, b\}^*$ for a CFG G over $\{a, b\}$ is undecidable.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
If A and B are regular and h a homomorphism then $h^{-1}(A \cap \sim B)$ is regular.
The halting problem for Turing machines is semi-decidable.
The grammar $S \to aS \mid a$ is strongly right-linear.
The regular expressions $a^*(a+b)^*b$ and $(a+b)(a^*+b)^*$ are equivalent.
There is a Turing machine that, given input of length n, halts if and only if the n-th Fibonacci number is even.
If A is recursive then so is $\sim A$.
Every set which can be proved non-context-free using the pumping lemma can also be proved non-context-free using Ogden's lemma.
Every Kleene algebra satisfies $a \leqslant a + b$.
The PCP instance $(10, 01)$, $(101, 110)$, $(1, 11)$, $(010, 00)$ has a solution.
Every set generated by an unrestricted grammar is also generated by a context-free grammar.