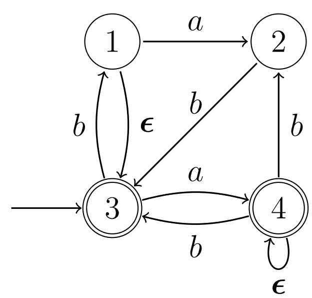
Formal Language and Automata Theory WS 2015/2016 LVA 703603

EXAM 1 February 1, 2016

This exam consists of <u>five</u> exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

$\boxed{1}$ Consider the regular expression $\alpha = (a(a+ba)^*+\boldsymbol{\epsilon})b^*$, the homomorphism $h\colon \{a,b\}^* \to \{a,b\}^*$ defined by $h(a) = \epsilon$, $h(b) = ab$, and the finite automaton N:



[10] (a) Compute $h((\alpha_a)_b)$.

[10] (b) Construct a DFA M such that $L(M) = {\sim}L(N) \cap h^{-1}(\alpha_b)$.

$\boxed{2}$ Consider the following context-free grammar G over $\Sigma = \{a,b\}$:

$$S \to AB \mid BA \qquad A \to CAC \mid ab \qquad B \to CBC \mid ba \qquad C \to a \mid b$$

[4] (a) Show that the grammar G is ambiguous.

[9] (b) Transform G into an equivalent grammar in Chomsky normal form and use the CKY algorithm to determine whether the string $bababa$ belongs to $L(G)$.

[7] (c) Construct an NPDA over $\Sigma = \{a,b\}$ that accepts the set

$$D = \{axby, bxay \mid x,y \in \Sigma^* \text{ and } |x| = |y|\}$$

of concatenations of two strings of the same length having distinct initial letters.

$\boxed{3}$ Determine whether the following sets over $\Sigma = \{a,b,c\}$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free. Prove your answers.

[6] (a) $A = \{x \in \Sigma^* \mid \#a(x) > 4\}$

[7] (b) $B = \{a^n b^m a^n b^m \mid n,m \geqslant 0\}$

[7] (c) $C = \{a^n b^m c^{n+m} \mid n,m \geqslant 0\}$

[10] $\boxed{4}$ (a) Recall the formal definition of (many-one) reduction and state the two main theorems concerning reduction.

[10] (b) Consider the following decision problem P:

input:	Turing machine M
question:	Is $L(M)$ regular?

Use a reduction, e.g. from the halting problem for Turing machines, to prove that P is undecidable.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
If $A \leq_{\mathsf{m}} \mathsf{HP}$ then $A \leq_{\mathsf{m}} \mathsf{TOTAL}$.
The set $\{a^n \mid n \text{ is a prime number}\}$ is context-free but not regular.
Every regular set is generated by a non-ambiguous context-free grammar.
Every NPDA can be transformed into a DPDA that accepts the same set.
The set $h^{-1}(h(L))$ is context-free for every context-free set L and homomorphism h.
The intersection of a context-free set with the complement of a finite set of words is regular.
For every NPDA M that accepts L by final state there exists an NPDA that accepts $\sim L$ by empty stack.
For every context-sensitive grammar G over Σ with start symbol S, if $x, y, z \in \Sigma^*$ then $S \xrightarrow[G]{*} y$ if and only if $xSz \xrightarrow[G]{*} xyz$.
Given two regular expression x and y over $\Sigma = \{a, b\}$, it is decidable whether their respective derivatives x_a and y_b are equivalent, i.e., whether $L(x_a) = L(x_b)$.
The diagonalization proof method may be used to show that in an enumeration of all Turing machines there must be some Turing machine that decides the halting problem.