Formal Language and Automata Theory WS 2015/2016 LVA 703603

EXAM 3 September 26, 2016

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

$\boxed{1}$ Consider sets over $\Sigma = \{0, 1, 2\}$ and let $\gamma = (0+2)(0+2)^*$. Define the *reversal* of regular expressions inductively by:

$$\texttt{rev}(a) = a \qquad \texttt{rev}(\varnothing) = \varnothing \qquad \texttt{rev}(\epsilon) = \epsilon$$
$$\texttt{rev}(\alpha + \beta) = \texttt{rev}(\beta) + \texttt{rev}(\alpha) \qquad \texttt{rev}(\alpha\beta) = \texttt{rev}(\beta)\,\texttt{rev}(\alpha) \qquad \texttt{rev}(\alpha^*) = \texttt{rev}(\alpha)^*$$

We define the reversal of a set A of strings over Σ as $\texttt{rev}(A) = \{\texttt{rev}(x) \mid x \in A\}$. (We have $\texttt{rev}(\gamma) = (2+0)^*(2+0)$ and $\texttt{rev}(\{0121, 10\}) = \{1210, 01\}$.)

[9] (a) Give examples of strings both in and not in $L(\gamma)$ and prove that $L(\gamma) = L(\texttt{rev}(\gamma))$ using

i. minimal DFAs,

ii. derivatives, or

iii. Kleene algebra.

Clearly indicate which of these three methods you choose.

[5] (b) Construct a regular expression α such that $\alpha = \texttt{rev}(\alpha)$ and $L(\alpha)$ contains a string that is not a palindrome.

[6] (c) Prove that $\texttt{rev}(L(\alpha)) = L(\texttt{rev}(\alpha))$ for every regular expression α. (You may use that $\texttt{rev}(AB) = \texttt{rev}(B)\,\texttt{rev}(A)$ and $\texttt{rev}(A^*) = \texttt{rev}(A)^*$ for sets A and B of strings.)

$\boxed{2}$ Consider the following context-free grammar G over $\Sigma = \{a, b, c, d\}$:

$$S \to aS \mid aSbS \mid T \qquad T \to c \mid dTT$$

[14] (a) Compute a Chomsky normal form G_1 of G and use the CKY algorithm to determine whether $acbdcc \in L(G_1)$.

[6] (b) Show that G is ambiguous.

$\boxed{3}$ Determine whether the following sets over $\Sigma = \{0, 1, 2\}$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free. Prove your answers.

[7] (a) $A = \{0x1y2 \mid x, y \in \Sigma^* \text{ and } x = \textbf{rev}\ y\}$

[7] (b) $B = \{x \in \Sigma^* \mid \#0(x) = \#1(xx) = \#2(xxx)\}$

[6] (c) $C = A \cup (\sim B)^*$

[10] $\boxed{4}$ (a) Show that the problem whether a Turing machine M on input x ever attempts to move its head left at any point during computation on x, is decidable.

[10] (b) Show that the problem whether a Turing machine M on input x ever attempts to write a blank at the beginning of the tape, right next to its left endmarker, is undecidable.

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$\mathsf{TOTAL} \leq_\mathsf{m} \mathsf{MP}$
Any finite set of strings is regular.
In Kleene algebra, $ab \leqslant bc$ implies $a^*b \leqslant bc^*$.
It is decidable whether a regular set is empty.
The set $\{p \mid p \text{ is prime}\}$ is ultimately periodic.
If $A \leq_\mathsf{m} B$ and A is decidable then B is decidable.
The intersection of two regular sets is context-free.
The derivatives of a context-free set are context-free.
If A and $\sim A$ are recursively enumerable then $\sim A$ is recursive.
Any context-free set is accepted by a deterministic push-down automaton.