EXAM 3 September 28, 2017

This exam consists of five exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

[10] $\boxed{1}$ (a) Give three different proofs which show that the set $A = \{a^{n^3} \mid n \geqslant 0\}$ is nonregular.

[10] (b) Determine whether the regular expression $\alpha = (a+b)^*a(a+b)(a+b)$ and the NFA N

		a	b
$\rightarrow$	1	$\{1,2\}$	$\{1,3\}$
	2	$\{3,5\}$	$\{5\}$
	3	$\varnothing$	$\{3,4\}$
$\rightarrow$	4	$\{4\}$	$\{3\}$
	5	$\{4,6\}$	$\{6\}$
	$6F$	$\{3,4\}$	$\varnothing$

are equivalent.

[7] $\boxed{2}$ (a) Complete the transition function δ such that the NPDA

$$N = (\{1,2\}, \{a,b\}, \{A, B, \perp\}, \delta, 1, \perp)$$

accepts the set $A = \{xb(\mathbf{rev}\ x) \mid x \in \{a,b\}^*\}$ by empty stack. Here δ consists of the following pairs:

$$\begin{array}{lll}
((1,a,\perp),(1,A\perp)) & ((1,a,A),\boxed{}) & ((1,a,B),\boxed{}) \\
((1,b,\perp),\boxed{}) & ((1,b,A),\boxed{}) & ((1,b,B),(1,BB)) \\
((1,b,\perp),(2,\perp)) & ((1,b,A),\boxed{}) & ((1,b,B),\boxed{}) \\
((2,a,A),(2,\epsilon)) & (\boxed{},\boxed{}) & (\boxed{},\boxed{})
\end{array}$$

[9] (b) Compute a CFG G such that $L(G) = {\sim}A \cup h(A) \cup h^{-1}(A)$ where h is the homomorphism defined by $h(a) = bb$ and $h(b) = a$.

[4] (c) Construct an NPDA M such that $L_f(M) = L_e(N)$.

$\boxed{3}$ Determine whether the following sets over $\Sigma = \{a,b\}$ are (i) regular, (ii) context-free but not regular, or (iii) not context-free. Prove your answers.

[7] (a) $A = \{xxx \mid x \in \{a\}^*\}$

[6] (b) $B = \{xxx \mid x \in \{a,b\}^*\}$

[7] (c) $C = {\sim}A$

$\boxed{4}$ Let

$$\mathsf{NO\_A} = \{M \mid M \text{ is a TM which never writes an } a \text{ on its tape}\}$$
$$\mathsf{MAX100} = \{M \mid M \text{ is a TM which accepts some string in 100 steps}\}$$

[7] (a) Use a reduction to show that NO_A is not recursively enumerable.

[7] (b) Is MAX100 recursive? Give a proof.

[6] (c) Let $A \subseteq \{0,1\}^*$ such that $A \leq_{\mathsf{m}} L(0^*1^*)$. Is $\sim A$ recursive?

[20] $\boxed{5}$ Determine whether the following statements are true or false. Every correct answer is worth 2 points. For every wrong answer 1 point is subtracted, provided the total number of points is non-negative.

statement
$H((a^* + b)^*((a^*)^*)^*) = 2$
The intersection of recursive sets is recursive.
Post correspondence problem is semi-decidable.
The identity $a + a^* = a^*$ holds in every Kleene algebra.
$((aa + b + \boldsymbol{\epsilon})^*(a + \epsilon)a^*)_a \equiv a(aa + b + \boldsymbol{\epsilon})^*(a + \epsilon)a^* + a^*$
Every non-trivial property of recursive sets is undecidable.
Ambiguity is a decidable property of context-free grammars.
Every regular set is generated by a strongly right-linear context-free grammar.
The class of deterministic context-free sets is closed under homomorphic image.
The set VALCOMPS(M, x) is context-free for every Turing machine M and input string x.