All Solutions

2 We take $\mathsf{boolToInt} = \lambda b.\,\mathsf{ite}\ b\ 1\ 0$ and compute the normal form of $\mathsf{boolToInt\ True}$ as follows:

$$\begin{aligned}
\mathsf{boolToInt\ True} &= \underline{(\lambda b.\,\mathsf{ite}\ b\ 1\ 0)\ \mathsf{True}} \\
&\to_\beta \mathsf{ite\ True}\ 1\ 0 = \underline{(\lambda xyz.\,x\ y\ z)\ \mathsf{True}}\ 1\ 0 \\
&\to_\beta \underline{(\lambda yz.\,\mathsf{True}\ y\ z)\ 1}\ 0 \\
&\to_\beta \underline{(\lambda z.\,\mathsf{True}\ 1\ z)\ 0} \\
&\to_\beta \mathsf{True}\ 1\ 0 = \underline{(\lambda xy.\,x)\ 1}\ 0 \\
&\to_\beta (\lambda y.\,1)\ 0 \\
&\to_\beta 1
\end{aligned}$$

3 Since $E = A\ A$, we want the following property to hold

$$A\ A\ t \to_\beta^* A\ A.$$

This tells us that A is a "function" that takes two arguments (A and t), duplicates the first one (A) and ignores the second one (t). Thus a possible solution is $A = \lambda ax.\,a\ a$, which is shown by the following β-reduction:

$$\begin{aligned}
E\ t &= A\ A\ t \\
&= \underline{(\lambda ax.\,a\ a)\ A}\ t \\
&\to_\beta \underline{(\lambda x.\,A\ A)\ t} \\
&\to_\beta A\ A = E
\end{aligned}$$