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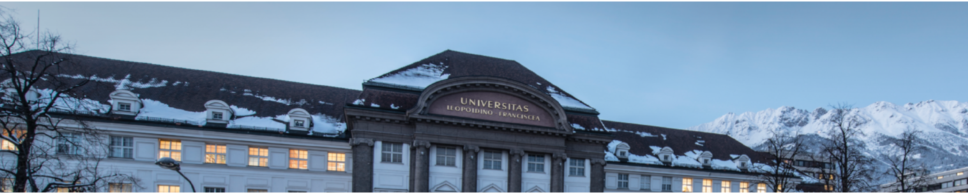

Functional Programming

Lecture 2

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Topics

abstract data types, algebraic data types, binary search trees, combinator parsing, efficiency, encoding data types as lambda-terms, evaluation strategies, formal verification, first steps, guarded recursion, Haskell introduction, higher-order functions, historical overview, induction, infinite data structures, input and output, lambda-calculus, lazy evaluation, list comprehensions, lists, modules, pattern matching, polymorphism, property-based testing, reasoning about functional programs, recursive functions, sets, strings, tail recursion, trees, tupling, type checking, type inference, types, types and type classes, unification, user-defined types

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Overview

- Types and Type Classes
- Lists
- Patterns, Guards, and More
- Higher-Order Functions

Types and Type Classes

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- type signature/constraint e `::` τ means "e is of type τ"

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syntactic sugar for list of characters, e.g., `['a','b','c']`

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Note – Show Types in GHCi

- `Prelude> :set +t`
- commands may be put inside `~/.ghci` (read on GHCi startup)

List Types

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- length: 2 (pair), 3 (triple), 4 (quadruple), ..., n (n-tuple), ...
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- fixed number of elements

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Examples

```
['a','b','c','d'] :: [Char]
["One","Two","Three"] :: [String]
[['a','b'],['c','d','e']] :: [[Char]]
(False,True) :: (Bool,Bool)
("Yes",True,'a') :: (String,Bool,Char)
```

Function Types

- type of functions from values of type τ_1 to values of type τ_2: τ_1 `->` τ_2
- every function takes single argument and returns single result
- simulating multiple arguments: use tuples

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Examples

```
not :: Bool -> Bool
not True = False
not False = True

add :: (Int, Int) -> Int
add (x, y) = x + y
```

Currying

- transform function taking tuple as input into function returning another function as output
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Examples

```
not True      == False
(False && x) == False
(True || x)  == True
otherwise     == True

fst (x, y) == x
snd (x, y) == y
```

Overloaded Types

- support standard set of operations
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Realization – Class Constrains

- syntax: e :: C `a` => τ
- meaning: "for every type `a` of class C, the type of e is τ"
  (where τ does contain `a`)

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Example – Addition

- `(+) :: Num a => a -> a -> a`
- "for every type `a` of class `Num`, addition has type `a -> a -> a`"
- since, e.g., `Int` is of class `Num`, we obtain that addition is of type `Int -> Int -> Int`, when used on `Int`s

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Example – Addition

(op) turns infix op into prefix

- `(+) :: Num a => a -> a -> a`
- "for every type a of class Num, addition has type `a -> a -> a`"
- since, e.g., Int is of class Num, we obtain that addition is of type `Int -> Int -> Int`, when used on Ints

The `Eq` Class – Equality

- specification, one of:

```
(==) :: Eq a => a -> a -> Bool
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The `Ord` Class – Orders

- prerequisite: `Eq`
- specification, one of:
```
compare :: Ord a => a -> a -> Ordering
(<=)    :: Ord a => a -> a -> Bool
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- where `Ordering` = {`LT`, `EQ`, `GT`}
- additional functions: `(<)`, `(>=)`, `(>)`, `min`, `max`

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The `Read` Class – "from string"

- useful functions:
```
read :: Read a => String -> a
```

The `Show` Class – "to string"

- specification, one of:

```
show     :: Show a => a -> String
showsPrec:: Show a => Int -> a -> String -> String
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- additional functions: `showList`

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The `Num` Class – Numeric Types

- prerequisites: `Eq` and `Show`
- specification, all of:

```
(+)         :: Num a => a -> a -> a
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(-)         :: Num a => a -> a -> a
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- additional functions: `negate`

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visit: `http://haskell.org` → `Documentation` → `Language Report: Haskell 2010`

Lists

Constructing Lists

- `[a]` ::= `[]` | `a : [a]`
- for given list, exactly two cases: either empty `[]`, or contains at least one element x and a remaining list xs (x : xs)
- `[`x_1`,`x_2`,` ... `,`x_n`]` abbreviates x_1 : (x_2 : ($\cdots$: (x_n : `[]`) $\cdots$))
- `(:)` is right-associative, hence x_1 : (x_2 : xs) = x_1 : x_2 : xs

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Examples

```
1 : (2 : (3 : (4 : []))) == 1 : 2 : 3 : 4 : []
1 : 2 : 3 : 4 : []       == [1,2,3,4]
1 : [2,3,4]              == [1,2,3,4]
```

Accessing List Elements – Selectors

- `head :: [a] -> a` – extract first element (fail on empty list)
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A Polymorphic List Function

- polymorphic means "having many forms"
- definition

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myReplicate n x =
  if n <= 0 then []
  else x : myReplicate (n - 1) x
```

- `myReplicate` has type `(Ord t, Num t) => t -> a -> [a]`
- can construct lists with elements of arbitrary type `a`, where length is given by some ordered numeric type `t`

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Exercise

use equational reasoning to evaluate `myReplicate 2 'c'`

Testing for Emptiness

- `null :: [a] -> Bool` – `True` iff argument is empty list

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Functions on Integer Lists

```
range m n =
  if m > n then []
  else m : range (m + 1) n

mySum xs =
  if null xs then 0
  else head xs + mySum (tail xs)

prod xs =
  if null xs then 1
  else head xs * prod (tail xs)
```

Examples

```
range 1 3 = [1,2,3]
range 3 2 = []

mySum [1,2,3] = 1 + 2 + 3 + 0
mySum [] = 0

prod [1,2,3] = 1 * 2 * 3 * 1
prod [] = 1
```

$$\texttt{mySum (range 1 n)} = \sum_{i=1}^{\texttt{n}} i$$

Patterns, Guards, and More

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	|	`x`	variable pattern
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	|	[$\langle pat \rangle$, ... , $\langle pat \rangle$]	list pattern
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	|	`C` $\langle pat \rangle$ ... $\langle pat \rangle$	constructor pattern

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- constructor patterns match the described application of a data constructor (example constructors are `(:)` and `[]` for lists, `True` and `False` for Boolean values, . . .)
- patterns may be used in arguments of function definitions and together with the **`case`** construct

The `case` Construct

$$
\begin{aligned}
\texttt{case}\ e\ \texttt{of}\ \langle pat_1\rangle \quad & \texttt{->} \quad e_1 \\
\vdots \qquad & \\
\langle pat_n\rangle \quad & \texttt{->} \quad e_n
\end{aligned}
$$

- checks $\langle pat_1\rangle$ to $\langle pat_n\rangle$ top to bottom
- if $\langle pat_i\rangle$ is first match, e_i is evaluated

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Example – Pattern Matching

```
mySum [] = ... -- constructor pattern
fst (x, _) = x -- patterns: tuple, variable, wildcard
case xs of [x] -> ... -- patterns: list, variable
           _   -> ... -- wildcard
```

Pattern Guards

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Example

```
f1 (x, _) | x >= 0  = x -- only if x non-negative
f2 (x:xs) | null xs = ... -- same as [x]
```

Refined Definitions

```
myReplicate n x | n <= 0    = []
                | otherwise = x : myReplicate (n - 1) x

range m n | m > n     = []
          | otherwise = m : range (m + 1) n

mySum []     = 0
mySum (x:xs) = x + mySum xs

prod []     = 1
prod (x:xs) = x * prod xs
```

Higher-Order Functions

Definition

a function is of higher-order if it takes functions as arguments

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Examples

```
twice f x = f (f x) -- apply f twice to x
```

Definition

a function is of higher-order if it takes functions as arguments

Examples

```
twice f x = f (f x) -- apply f twice to x
```

Sections

- abbreviation for partially applied infix operators
- `(x `op`)` abbreviates `(\y -> x `op` y)`
- `(`op` y)` abbreviates `(\x -> x `op` y)`

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Examples

```
ghci> twice (^2) 10
10000
```

Processing Lists – `map`

- possible definition

```
map :: (a -> b) -> [a] -> [b]
map f []      = []
map f (x:xs) = f x : map f xs
```

- syntactic sugar `map f xs` = `[f x | x <- xs]`

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Examples

```
ghci> map (+1) [1,3,5,7]
[2,4,6,8]
ghci> import Data.Char
ghci> map isDigit ['a','1','b','2']
[False,True,False,True]
ghci> map reverse ["abc","def","ghi"]
["cba","fed","ihg"]
ghci> map (map (+1)) [[1,2,3],[4,5]]
[[2,3,4],[5,6]]
```

Processing Lists – `filter`

- possible definition

```
filter :: (a -> Bool) -> [a] -> [a]
filter p []    = []
filter p (x:xs)
  | p x        = x : filter p xs
  | otherwise = filter p xs
```

- syntactic sugar `filter p xs` = `[x | x <- xs, p x]`

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Examples

```
ghci> filter even [1..10]
[2,4,6,8,10]
ghci> filter (>5) [1..10]
[6,7,8,9,10]
ghci> filter (/= ' ') "abc def ghi"
"abcdefghi"
```

“Fold Right” – A Very Expressive Function

- possible definition

```
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f b []       = b
foldr f b (x:xs) = x `f` (foldr f b xs)
```

- `b` is ‘base value‘
- `f` combining function (binary)
- intuitively `foldr f b` $[x_1, x_2, \ldots, x_n]$

$$= \texttt{foldr f b} \; (x_1 \;\texttt{:}\; (x_2 \;\texttt{:}\; \cdots \; (x_n \;\texttt{:}\; \texttt{[]}) \cdots))$$
$$= (x_1 \;\texttt{`f`}\; (x_2 \;\texttt{`f`}\; \ldots \; (x_n \;\texttt{`f`}\; \texttt{b}\,) \ldots))$$

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- f combining function (binary)
- intuitively foldr f b $[x_1, x_2, \ldots, x_n]$

$$
\begin{aligned}
&= \texttt{foldr f b } (x_1 \ \texttt{:}\ (x_2 \ \texttt{:}\ \cdots \ (x_n \ \texttt{:}\ \texttt{[]})\cdots)) \\
&= (x_1 \ \texttt{`f`}\ (x_2 \ \texttt{`f`}\ \ldots \ (x_n \ \texttt{`f`}\ \texttt{b})\ldots))
\end{aligned}
$$

This Pattern is Very General

- take (+) for f and 0 for b: foldr (+) 0 = sum
- take (*) for f and 1 for b: foldr (*) 1 = product
- take const (+1) for f and 0 for b:
  foldr (const (+1)) 0 = length (where const f _ = f)

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- b is 'base value'
- f combining function (binary)
- intuitively foldr f b [x_1,x_2,...,x_n]

$$= \texttt{foldr f b} \ (x_1 \ : \ (x_2 \ : \ \cdots \ (x_n \ : \ \texttt{[]}) \cdots))$$
$$= (x_1 \ \texttt{`f`} \ (x_2 \ \texttt{`f`} \ \ldots \ (x_n \ \texttt{`f`} \ \texttt{b}) \ldots))$$

This Pattern is Very General

add dummy argument

- take (+) for f and 0 for b: foldr (+) 0 = sum
- take (*) for f and 1 for b: foldr (*) 1 = product
- take const (+1) for f and 0 for b:
  foldr (const (+1)) 0 = length (where const f _ = f)

Homework (for November 9th)

1. Read Chapters 1 and 2 of Real World Haskell.
2. Work through lessons 4 to 5 on `http://tryhaskell.org/`.
3. Give the types (and class constraints) for each of:

```
pair x y          =  (x, y)
tail2 xs          =  tail (tail xs)
triple x          =  x * 3
thrice f x        =  f (f (f x))
mapPair f (x, y)  =  (f x, f y)
idList            =  filter (const True)
```

4. Use equational reasoning to stepwise compute the result of `filter (const False) ["a","b","c"]` on paper.
5. Using `foldr`, give alternative definitions of two of the functions we have seen so far (excluding those that we already defined via `foldr`).
6. Define a function `intercalate :: [a] -> [[a]] -> [a]` such that `intercalate xs xss` inserts the list `xs` between the lists in `xss` and concatenates the result.

   Example:

   `intercalate "; " ["one","two","six"]` = `"one; two; six"`