universität innsbruck

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Functional Programming

Lecture 11

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Topics

abstract data types, algebraic data types, binary search trees, combinator parsing, efficiency, encoding data types as lambda-terms, evaluation strategies, formal verification, first steps, guarded recursion, Haskell introduction, higher-order functions, historical overview, induction, infinite data structures, input and output, lambda-calculus, lazy evaluation, list comprehensions, lists, modules, pattern matching, polymorphism, property-based testing, reasoning about functional programs, recursive functions, sets, strings, tail recursion, trees, tupling, type checking, type inference, types, types and type classes, unification, user-defined types

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Overview

- Laziness and Infinite Data Structures
- Course Evaluation
- Examples of (Infinite) Laziness
- Recommendations

Laziness and Infinite Data Structures

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Example

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main = do
  let f1 x = x + 1
      f2 x = f2 x  -- a nonterminating function
  input <- getLine
  let i = read input
  print $ head [f1 i, f2 i]
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- value of `f2 i` not needed
- however, without laziness program would not terminate

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- (lazy) evaluation

```
fst . head . filter ((==1) . snd) $ zip [0..] [1..9]
  = ... $ (0,1) : zip [1..] [2..9]
  = fst . head $ (0,1) : filter ((==1) . snd) (...)
  = fst (0,1)
  = 0
```

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```

- without laziness (ignoring nontermination of `[0..]`) three list traversals (generation of `[0 .. length xs]`, `zip`, and `filter`)

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with laziness even tail recursive programs may run out of memory

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Function (Tail Recursive)

```
length' acc []       =  acc
length' acc (_:xs)   =  length' (acc + 1) xs
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Function (Tail Recursive)

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length' acc []       =  acc
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```

Evaluation

```
length' 0 [1,2,3,4]
  =  length' (0+1) [2,3,4]
  =  length' (0+1+1) [3,4]
  =  length' (0+1+1+1) [4]
  =  length' (0+1+1+1+1) []
  =  (0+1+1+1+1)
```

Being Strict – The seq Function

- type `seq :: a -> b -> b`
- `x `seq` y` reduces `x` to WHNF and returns `y`

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Strict Folding of Lists – Data.List.foldl'

```
foldl' _ b []     = b
foldl' f b (x:xs) = c `seq` foldl' f c xs
  where
    c = f b x
```

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  where
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```

Example

```
length'' = foldl' (const . (+1))
```

Course Evaluation

Evaluation

LVA-Code 703.024-0

Additional Questions

1. I am prepared to tackle moderate programming tasks using Haskell.
2. Having lecture notes (PDFs) in addition to slides is useful/important.
3. I would like to learn more about FP and/or use it for real world code.
4. The number of weekly exercises was appropriate.
   - *stimme völlig zu* = "should have been smaller"
   - *stimme gar nicht zu* = "should have been higher"

Examples of (Infinite) Laziness

Example 1 – Fibonacci Numbers (this time starting from 0)

$$\mathsf{fib}(i) = \begin{cases} 0 & \text{if } i \leq 0 \\ 1 & \text{if } i = 1 \\ \mathsf{fib}(i-1) + \mathsf{fib}(i-2) & \text{otherwise} \end{cases}$$

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Sequence

$0\ 1\ 1\ 2\ 3\ 5\ 8\ 13\ 21\ 34\ 55\ 89\ 144\ 233\ 377\ 610\ 987\ 1597\ 2584\ \ldots$

One Way of Computing Fibonacci Numbers

starting at 0 | 0 1

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1

One Way of Computing Fibonacci Numbers

starting at 0	0 1
starting at 1	1
$(+)$	

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One Way of Computing Fibonacci Numbers

starting at 0	0	1	1
starting at 1	1	1	
$(+)$	1		

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One Way of Computing Fibonacci Numbers

starting at 0	0	1	1
starting at 1	1	1	
$(+)$	1	2	

One Way of Computing Fibonacci Numbers

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One Way of Computing Fibonacci Numbers

starting at 0	0	1	1	2	3
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$(+)$	1	2	3	5	

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Ingredients

- function to shift sequence to left

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Ingredients

- function to shift sequence to left
- function to add two sequences

Fibonacci Numbers in Haskell

```haskell
fibs :: [Integer]
fibs = 0 : 1 : zipWith (+) fibs (tail fibs)
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Resulting Data Dependencies

```
[ 0 , 1 , _ , _ , _ , _ , _ , ... ]
```

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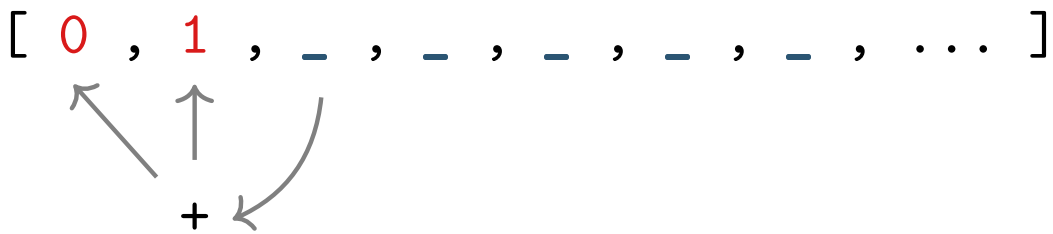
Resulting Data Dependencies



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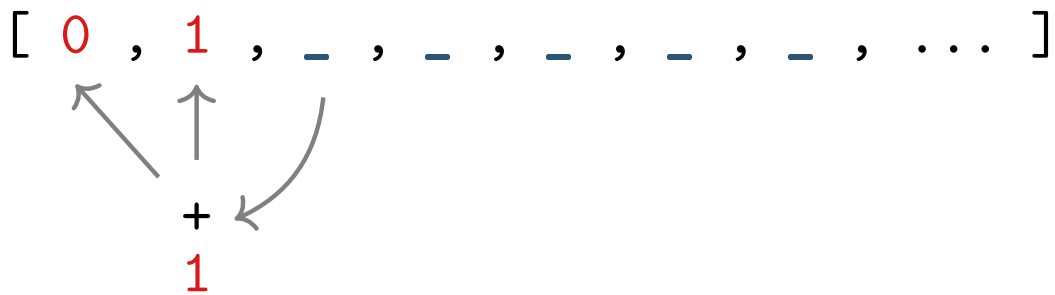

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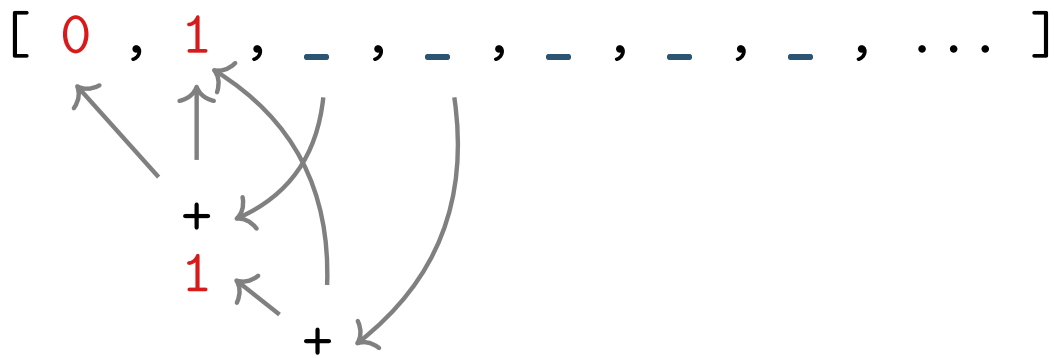

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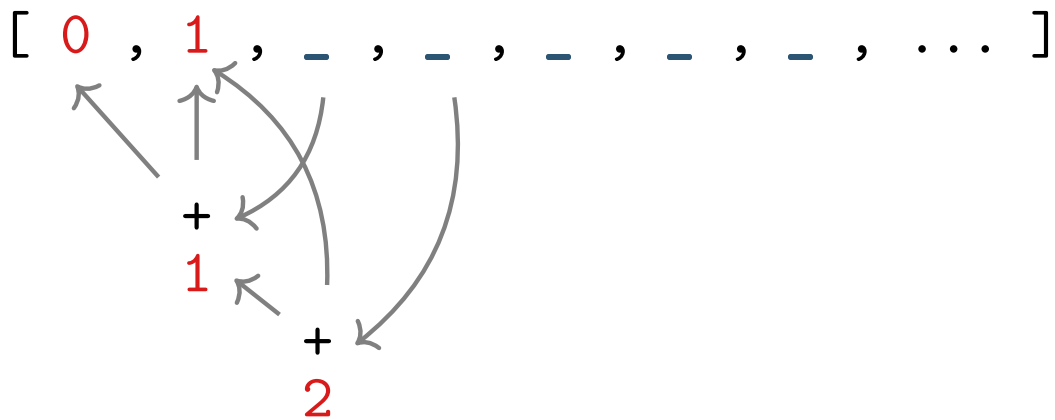

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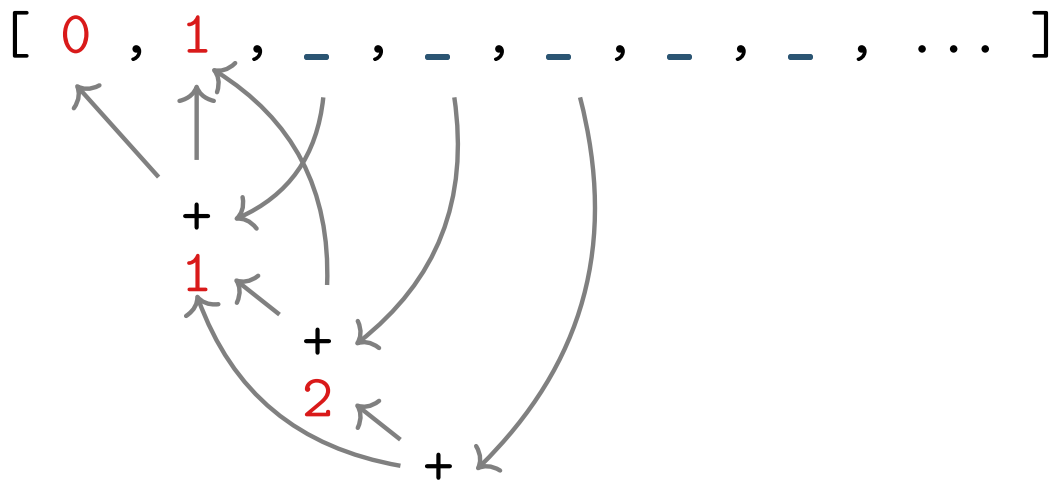

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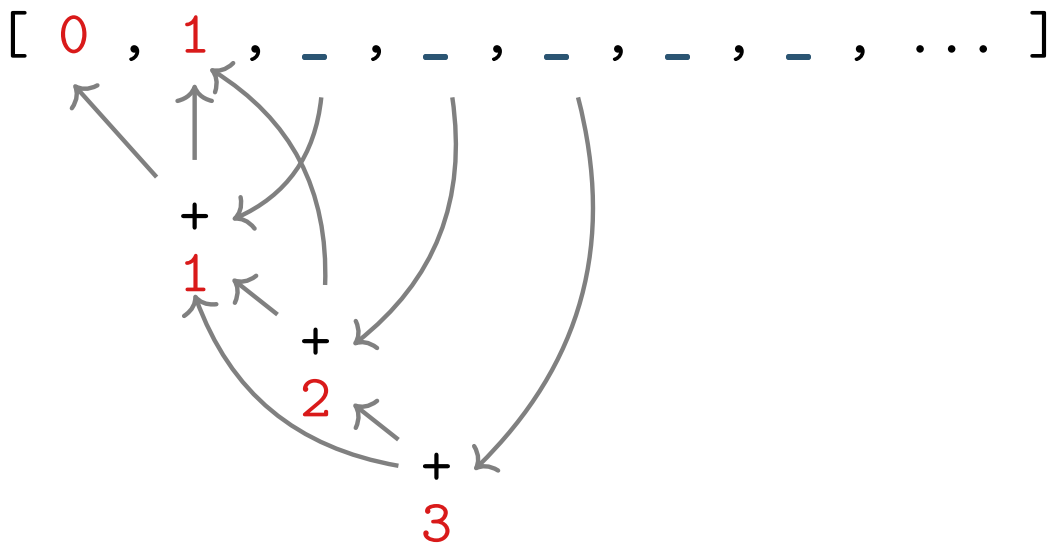

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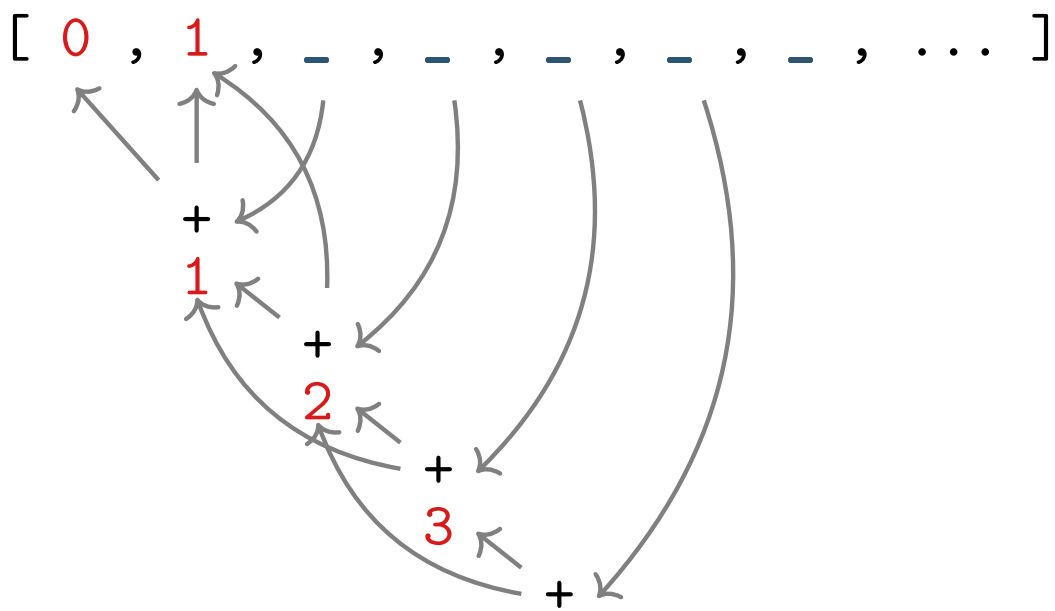

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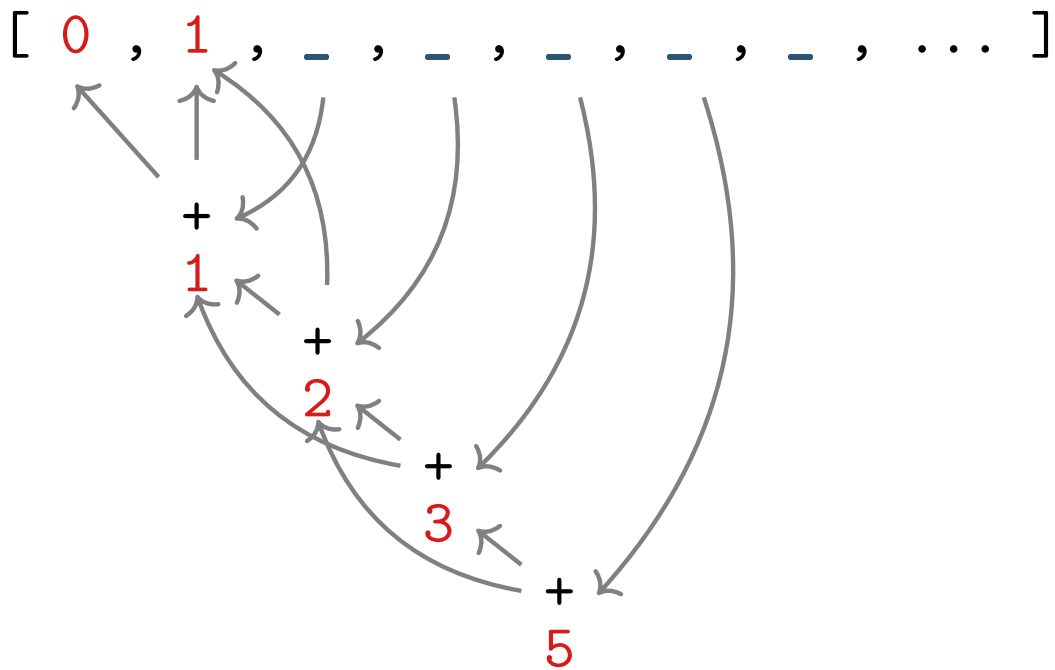

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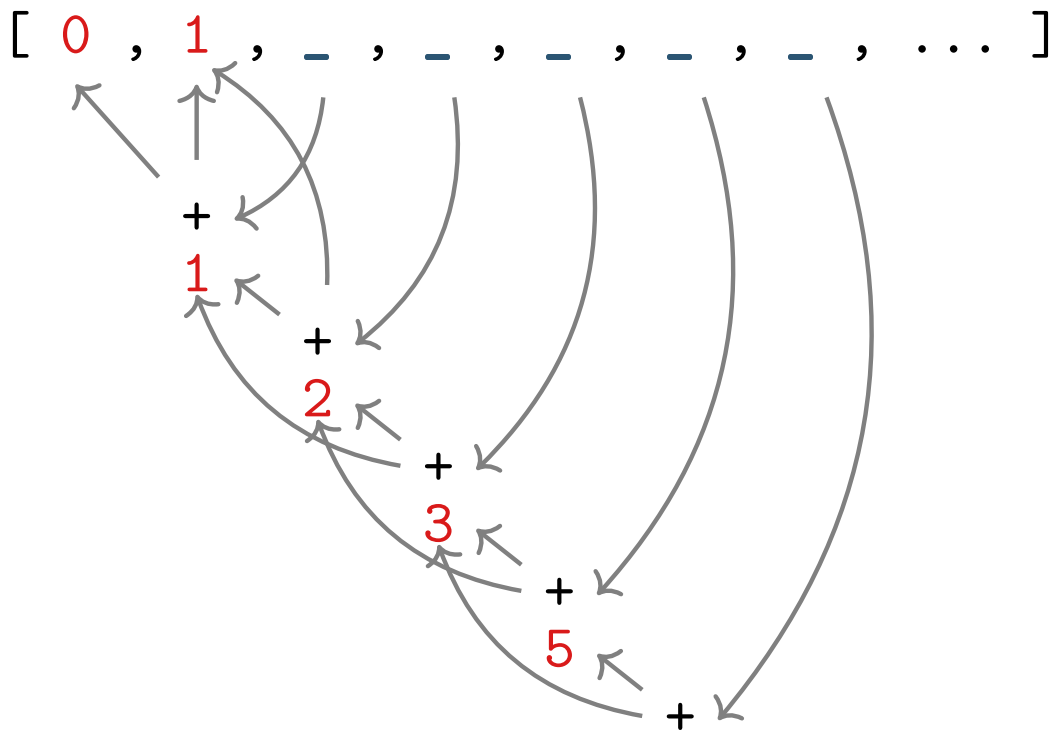

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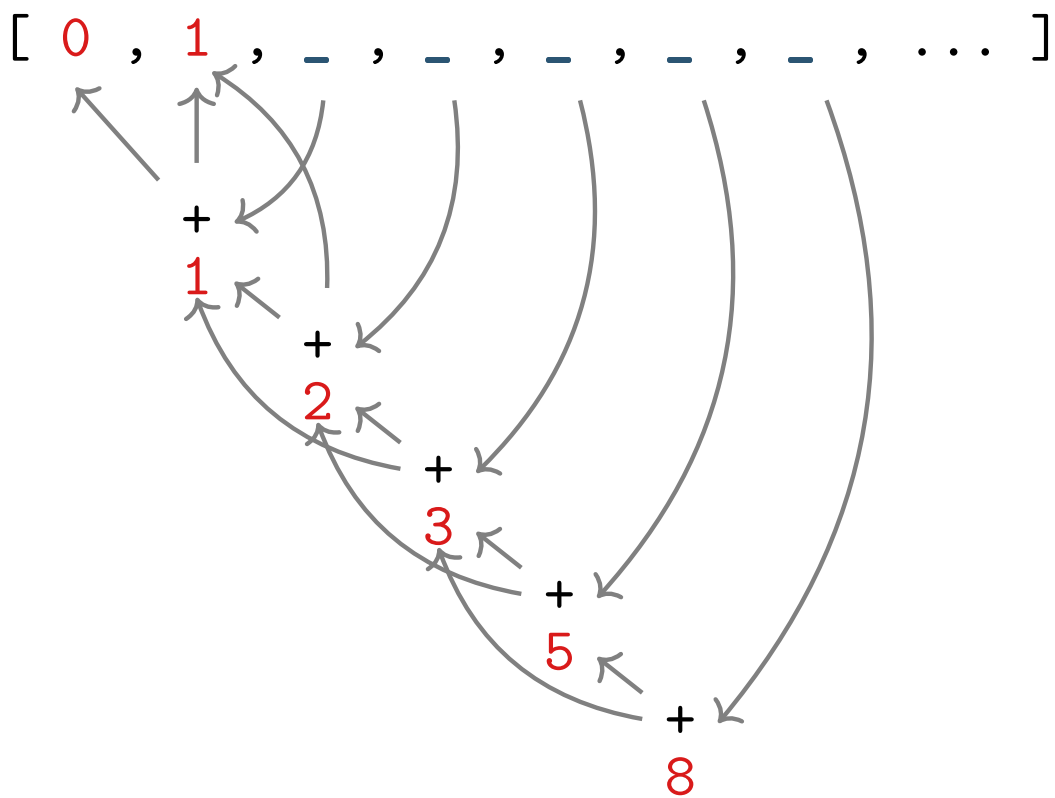

Example 2 – A Variation of the “Sieve of Eratosthenes”

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generate list of all prime numbers

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Procedure

start with list of all natural numbers (from 2 on)

1. mark first element x as prime
2. remove all multiples of x
3. go to Step 1

The Sieve in Haskell

```haskell
primes :: [Integer]
primes = sieve [2..]
  where
    sieve (x:xs) =
      x : sieve [y | y <- xs, y `mod` x /= 0]
```

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- `zipWith const` $[x_1, \ldots, x_m]$ $[y_1, \ldots, y_n] =$
  $[x_1, \ldots, x_{\min\{m,n\}}]$

Example 4 – Lazy Shenanigans

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```
replaceAllByMax xs = ys
  where
    (m, ys) = maxAndReplaceAll m xs
    maxAndReplaceAll c =
      foldr (\x (m, ys) -> (max x m, c : ys)) (0, [])
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result: (m , ys)

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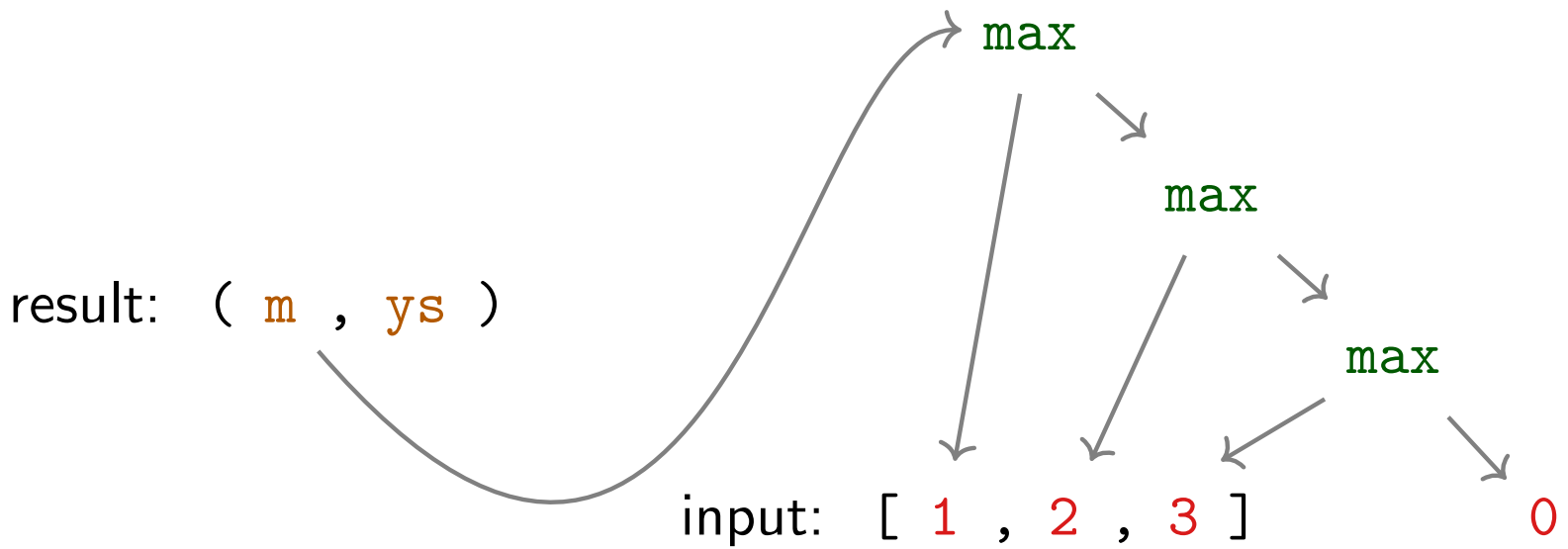

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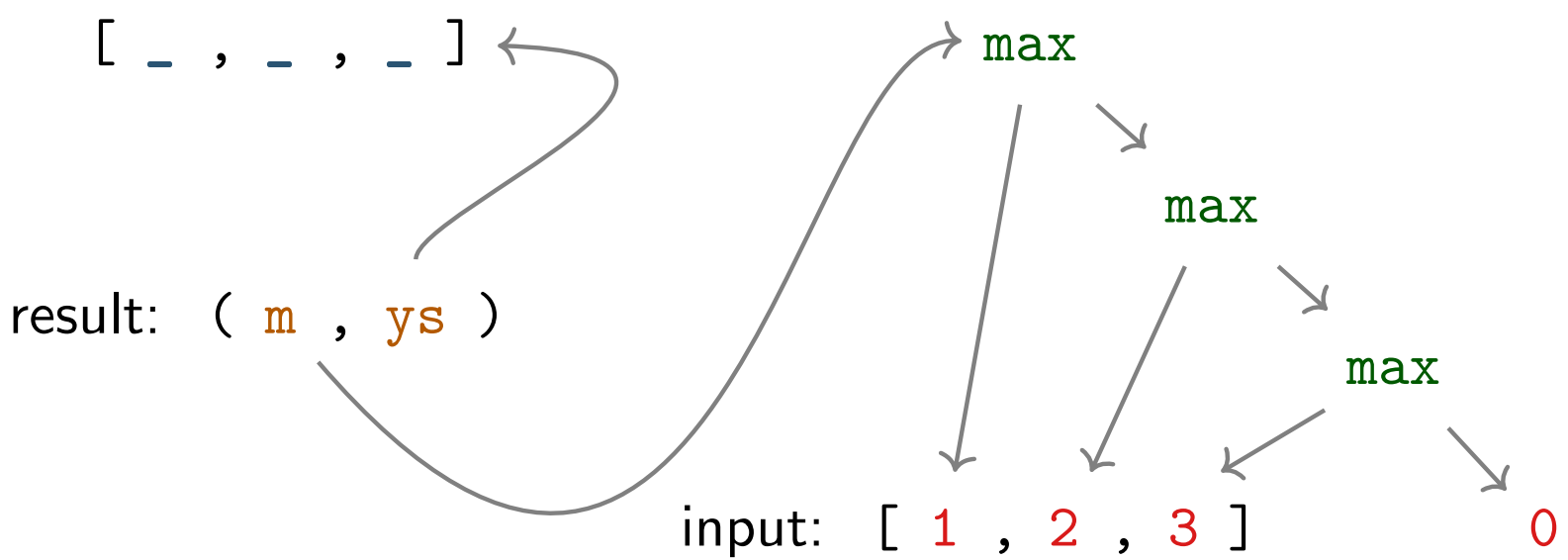

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      foldr (\x (m, ys) -> (max x m, c : ys)) (0, [])
```
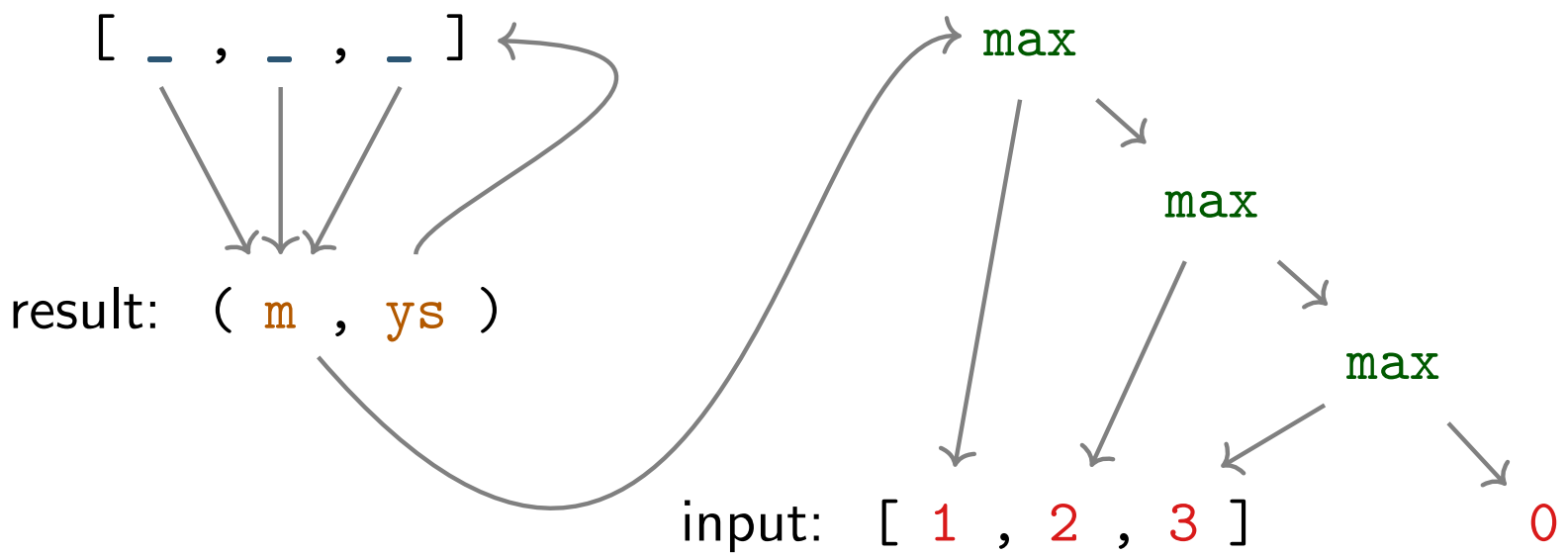

Resulting Data Dependencies for [1,2,3]



Recommendations

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- Term Rewriting (VO2 + PS1)

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- Meta-Tool Based Proof Search in Coq
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- Maximal Interpretations
- A Certified Decision Procedure for Termination of Right-Ground Term Rewrite Systems
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- ...