Starred exercises are optional.

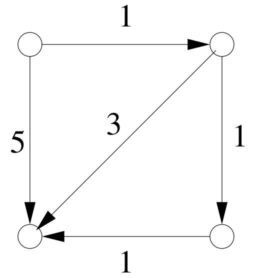
1) Let G be the directed graph given by the matrix $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

- Give a representation of G in terms of sets of vertices and edges, and draw G.
- Let B be the 'matrix product' of A with itself such that the elements of B are computed as $B_{ij} = \max_{k=1}^{4} A_{ik}A_{kj}$. For instance, for the element at the 2nd row and in the 4th column we compute $B_{2,4} = \max\{0 \cdot 0, 0 \cdot 0, 1 \cdot 1, 0 \cdot 0\} = 1$. Compute B and draw the corresponding graph.

  What is the meaning of (the edges of) the graph obtained by repeating this 'matrix product' n times in terms of G?
  Hint: Path

2)
- Suppose someone who is known to always take the shortest path (in a digraph), passes node c on his path from node a to node b. Is the path from a to c he took then also the shortest path from a to c? If so, show this (write down your reasoning as precisely as possible/needed to convince someone else). If not, give a counterexample.
- Argue that in a graph G having n nodes, if there is a path from node a to node b in G at all, then there is a path from a to b having length smaller than n.

3) Use Floyd's algorithm to compute the shortest paths between any two nodes in the weighted digraph G given by



in two ways: first by taking the top–right node for the first row/column of the matrix B, continuing clockwise, and next for a matrix B' obtained by starting with the bottom–right node. Do we obtain the same result, i.e. the same distances? Why (not)?

4∗)
- How many square n by n matrices whose elements are all 0 or 1, are there? How many digraphs having nodes $\{1, \dots, n\}$ are there?
- Give a program (in pseudocode or in some programming language) that generates all digraphs having nodes $\{1, \dots, n\}$, for arbitrary n.