Starred exercises are optional.

1. Consider the TM (in the format of `http://morphett.info/turing/turing.html`, so e.g. without explicit left-end marker) with input alphabet $\{0, 1\}$ and transition rules:

```
0 0 * r 0
0 1 * r 1
0 _ * r halt-accept
1 0 * r 2
1 1 * r 0
1 _ * r halt-reject
2 0 * r 1
2 1 * r 2
2 _ * r halt-reject
```

   Thinking of M as defining a property P of natural numbers, via their binary representations, M accepts 27 since it accepts its binary representation 11011, but rejects 14 as it rejects 1110.

   - What property P is defined by M? That is, give a description (in natural language) of the language $L(M)$ accepted by M. To find it, run M on the successive inputs 0, 1, 10, 11, 100, 101, 110, 111, 1000, and 1001 (corresponding to 1–9) and look for a pattern.
   - Is M a total TM? Is the language $L(M)$ recursive? Is the property P semi-decidable?
   - Construct a TM M' from M such that $L(M') = {\sim}L(M)$, i.e. such that M' accepts the complement of the language accepted by M.

2. Consider the TM K (for the format, see the previous exercise) having transition rules:

```
0 0 * r 0
0 1 * * 0
0 _ * l halt-accept
```

   Describe the language $L = L(K) \subseteq \{0, 1\}^*$ accepted by K, and show that the complement ${\sim}L = \{0, 1\}^* - L$ of L is recursive by providing an appropriate TM (in the same format).

3. Suppose $L_1 = L(M_1)$, $L_2 = L(M_2)$ for TMs M_1, M_2 with input alphabet $\{0, 1\}$. For each of the following 4 languages say whether it is recursively enumerable or not: 1) L_1 as language over $\{0, 1, 2\}$, 2) $L_1 \cup L_2$, 3) $L_1 \cap L_2$, 4) $L_1 - L_2$. For each item, in case your answer is affirmative indicate how an appropriate TM can be constructed, and in case it is negative explain why.

4∗ On slide 22 of lecture 12 it is checked that on inputs 10 and 001 the behaviour of the (hypothetical) TM CD is as given by cd. Complete this check, for the other inputs as far as specified by the matrix on slide 20.

   - If we were to change the top–right t into r in (the picture of) CD on slide 22, would the proof still go through?
   - Same question, but for changing the bottom-right ↻ into r?

5∗ Show that the membership problem MP is undecidable, by modifying the proof by diagonalisation in the slides of lecture 12 showing that the halting problem HP is undecidable.