Starred exercises are optional.

1) For the partial orders corresponding to the following strict orders R draw (an illustrative part of) its Hasse diagram, i.e. the smallest subrelation S (if it exists) with it as reflexive–transitive closure, and argue whether or not R is well-founded. Argue why there is (no) infinite descending chain. If there is, draw it. Otherwise, describe the minimal elements. (Updated 24-10)
   a) less-than $<$ on the natural numbers;
   b) greater-than $>$ on the natural numbers;
   c) the lexicographic order $<_{\text{lex}}$ on $\{a\}$;
   d) the lexicographic order $<_{\text{lex}}$ on $\{a, b\}$ with $a < b$;
   e) divisibility $\{(x, z) \mid x, z \in \mathbb{R}' \wedge \exists y \in \mathbb{R}', x \cdot y = z\}$ on the real numbers greater than 1, where $\mathbb{R}' = \{x \in \mathbb{R} \mid x > 1\}$;
   f) divisibility $\{(n, k) \mid n, k \in \mathbb{N}' \wedge \exists m \in \mathbb{N}', n \cdot m = k\}$ on the natural numbers greater than 1, where $\mathbb{N}' = \{n \in \mathbb{N}' \mid n > 1\}$.

2) For each of the following well-founded relations R_i on the set Σ^* of words over $\Sigma = \{0, 1\}$,
   - draw the graph of R_i for words up to (and including) length 3;
   - state the well-founded induction principle for R_i, considering *base* cases separately; and
   - describe the transitive closure R_i^+ in your own words.

   a) $R_1 = \{(w, aw) \mid a \in \Sigma, w \in \Sigma^*\}$;
   b) $R_2 = \{(w_1w_2, w_1aw_2) \mid a \in \Sigma, w_1, w_2 \in \Sigma^*\}$.
   c) $R_3 = \{(w, wa) \mid a \in \Sigma, w \in \Sigma^*\}$;

3) Prove one of the following (a $(*)$ for every extra one) by well-founded induction. Clearly state the well-founded relation R you use in your well-founded induction, and what are the induction hypotheses you use (for the cases considered).
   a) Prove that for all natural numbers n, $\sum_{i=1}^{n} 2^i = 2^{n+1} - 1$.
   b) Prove that every finite set A, has exactly $2^{|A|}$ subsets, where $|A|$ denotes the size of A, i.e. its number of elements (Updated 24-10).
   c) Prove that for all natural numbers n, k with $k \leq n$, $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ if $0 < k < n$ and 1 otherwise, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, the binomial coeffient.
   d) Prove that for all natural numbers x and $\leq$-sorted lists l of natural numbers, `bs x l` is `True` if x occurs in l, and `False` otherwise. (Look up `!!`, `take`, `drop` in a Haskell manual.)

```
bs x [] = False
bs x [y] = x == y
bs x l = if x < (l !! h) then bs x (take h l) else bs x (drop h l) where
  h = length l `div` 2
```

4∗) Show that taking the predecessor of the componentwise extension of a partial order is different from taking the componentwise extension of its predecessor.