Starred exercises are optional.

-) Exercise 3 of the previous sheet. (The crosses you already put, you keep; but everyone now has an opportunity to add to what he/she already had.)

1) Let the set W of words over $\Sigma = \{0, 1\}$ be inductively defined by the following four clauses:

   - $0110 \in W$;
   - $\epsilon \in W$;
   - if $w \in W$, then $0w1 \in W$;
   - if $w \in W$ then $1ww0 \in W$.

   That is, W is the *least* set satisfying these four clauses.

   - For the well-founded sub-word relation R_W on words in W induced by the clauses of this inductive definition (see slide 10 of week 5), draw the graph of R_W for words in W up to (and including) length 6;
   - Give the well-founded induction principle for words in W, corresponding to R_W;
   - Show, using that principle, that for all $w \in W$, w has the same number of 0s and 1s.

2) For 5 cases (your choice) in which the operations on relations (slide 17 of week 5 of the slides of the lecture) fail to preserve being a function (slide 18), a partial order (slide 19), or a well-founded relation (slide 20), as indicated by a cross behind them, give an example illustrating how preservation fails.

3) Show that the sets of integers $\mathbb{Z}$ and of natural numbers $\mathbb{N}$ are equinumerous by giving an appropriate bijection. The less-than-or-equal relation $\leq$ is *not* a well-order on $\mathbb{Z}$, but can you think of a well-order $\sqsubseteq$ on $\mathbb{Z}$ that *is* isomorphic to the less-than-or-equal well-order $\leq$ on $\mathbb{N}$? If so, give both the bijection and $\sqsubseteq$. Otherwise, explain why not.