Starred exercises are optional.

1. Let $A = \{a, b, c, d\}$, $B = \{\alpha, \beta\}$, $C = \{\alpha, \beta, c\}$ be sets. Let A and B be enumerated by respectively $e : \{0, 1, 2, 3\} \to A$ defined by $0 \mapsto c$, $1 \mapsto a$, $2 \mapsto b$, $3 \mapsto d$, and $f : \{0, 1\} \to B$ defined by $0 \mapsto \beta, 1 \mapsto \alpha$. For the constructions on slides 6 and 7 of lecture 7:
   a) Give the resulting numberings of A and $A \times B$, and their cardinalities.
   b) Give the resulting enumerations of $A \cup B$ and A^B, and their cardinalities.
   c) Compute $\#(A \cup B \cup C)$ using the inclusion/exclusion principle.

2. a) Suppose to have 17 pigeons and 5 pigeon holes. Prove that, if each pigeon goes into one of the pigeon holes, there must be a pigeon hole containing at least 4 pigeons.
   b) Consider the (single-elimination) knock-out tournament (e.g. like in tennis) having 16 players, whose *bracket* is given on `https://en.wikipedia.org/wiki/Bracket_(tournament)`.
      - Represent and draw the bracket as a tree (in the formal sense; completing outcomes of the final games as you wish), with players as leaves and games as non-leaf-nodes.
      - Give a natural bijection between the nodes of the players that did not win the tournament and the game nodes, and then use double-counting to conclude (*without* counting!) that the tournament must have 15 games.

   Generalise to conclude that in such tournaments for $n + 1$ players, n games are played.

3. Let A have 6 elements and B have 10 elements.
   a) How many functions $f : A \to B$ are there?
   b) How many *injective* functions $f : A \to B$?
   c) How many *bijective* functions $f : A \to B$?
   d) How many subsets does A have?
   e) How many subsets of size 7 does A have?

   If we would swap A and B, would any answer remain the same?

4*)
- Show $\max(n, m) = (n + m) - \min(n, m)$ for all natural numbers n, m. For instance, $\max(2, 7) = (2 + 7) - \min(2, 7) = 9 - 2 = 7$.
- Prove that for a collection $N = (n_i)_{1 \le i \le k}$ of k natural numbers n_i, and letting I range over subsets of $\{1, \ldots, k\}$:

$$\max N = \Big(\sum_{\#(I)\text{ odd}} \min_{i \in I} n_i\Big) - \Big(\sum_{\#(I)\text{ even, } I \text{ non-empty}} \min_{i \in I} n_i\Big)$$

  For instance, $\max(4, 2, 7) = (\min(4, 2, 7) + \min(4) + \min(2) + \min(7)) - (\min(4, 2) + \min(2, 7) + \min(7, 4)) = (2 + 4 + 2 + 7) - (2 + 2 + 4) = 15 - 8 = 7$.
  Hint: Cf. the inclusion/exclusion principle.

5*) (Added 15-11-2019) Let D be a domain having n elements.
- How many binary relations on D are there? How many nullary, unary, ternary?
- How many unary predicates (functions from D to booleans)? nullary, binary, ternary?