Starred exercises are optional.

1.
   - Show that there is a bijection between $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{>0}$, i.e. between the sets of non-negative, resp. positive real numbers.
   - Show that there is a bijection between $(0,1) \subseteq \mathbb{R}$ and $\mathbb{R}_{>0}$, i.e. between the set of real numbers (strictly) between 0 and 1, and the positive real numbers.
   - Show that $|\mathbb{N}| < |\mathbb{R} \times \mathbb{R}|$.

2.
   - Show that, for $n \geq 0$, the sets $B_n = \{w \mid w \in \Sigma^*, \ell(w) = n\}$ comprising strings of length n, partition the set of strings over $\Sigma = \{0,1\}$, and give the corresponding equivalence relation on those strings. What is the cardinality of B_n?
   - Check that the relation on positive fractions defined by $\frac{n}{m} = \frac{n'}{m'}$ if $n \cdot m' = m \cdot n'$, is an equivalence relation, and give a system of representatives.

3.
   - Compute the gcd of 42 and 63 both using the *subtraction*-based version and the *division*-based version of Euclid's algorithm.
   - Using the extended Euclid's algorithm (Bézout's lemma), compute u, v such that $u \cdot 77 + v \cdot 30 = 1$.

4∗) Let $f : \mathbb{N} \to \mathbb{Z}$ be defined by $f(n) = n$, and $g : \mathbb{Z} \to \mathbb{N}$ be defined by $g(0) = 0$, $g(x) = 2^x$ if $x > 0$, and 3^{-x} otherwise. Both f and g are injections but not bijections. However, by the (proof of the) theorem of Schröder–Bernstein (slides 11–12 of week 8) a bijection $f' : \mathbb{N} \to \mathbb{Z}$ with inverse $g' : \mathbb{Z} \to \mathbb{N}$ can be constructed from f and g. Describe (in words, by formulas, by drawing, or by other means), the functions f' and g' as constructed in that proof.

5∗) From slide 28 of lecture 8 we know that $\text{lcm}(n,m) = \frac{(n \cdot m)}{\gcd(m,n)}$ for all positive natural number n, m. For instance $\text{lcm}(15,24) = \frac{15 \cdot 24}{\gcd(15,24)} = \frac{360}{3} = 120$.

Argue that for a collection $N = (n_i)_{1 \leq i \leq k}$ of k positive natural numbers n_i, and letting I range over subsets of $\{1, \ldots, k\}$:

$$\text{lcm}\, N = \frac{(\prod_{\#(I)\text{ odd}} \gcd_{i \in I} n_i)}{(\prod_{\#(I)\text{ even}, \neq 0} \gcd_{i \in I} n_i)}$$

$$\text{e.g. } \text{lcm}(15,24,2) = \frac{\gcd(15,24,2) \cdot \gcd(15) \cdot \gcd(24) \cdot \gcd(2)}{\gcd(15,24) \cdot \gcd(24,2) \cdot \gcd(2,15)} = \frac{1 \cdot 15 \cdot 24 \cdot 2}{3 \cdot 2 \cdot 1} = \frac{720}{6} = 120$$

Hint: Cf. inclusion/exclusion