1) a) The Hasse diagram of $\leq$ is, see the figure below, the graph of the relation $S = \{(n, n+1) \mid n \in N\}$, for which it is readily verified that it is irreflexive and atransitive (if $x\ S\ y$ and $y\ S\ z$ then *not* $x\ S\ z$), $S^* = \leq$, and it is least: each $(n, n+1)$ must be in any sub-relation S' of $\leq$ such that $S'^* = \leq$ as $n < n+1$ and there is no k such that $n < k < n+1$. $<$ is well-founded as there is no descending chain longer than n from any n. 0 is minimal.

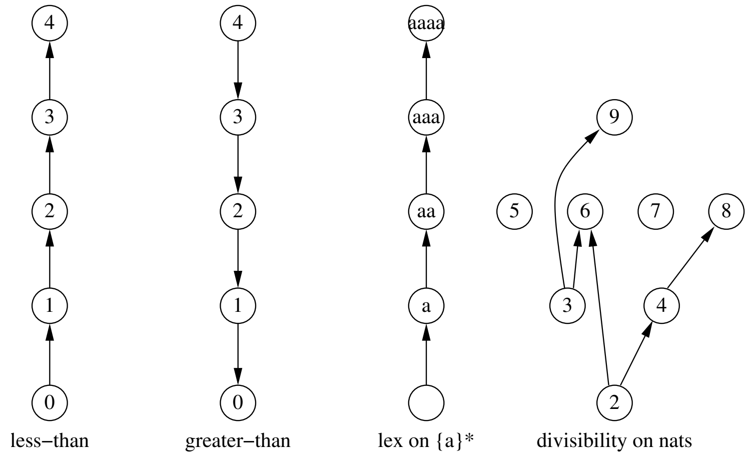
b) The Hasse diagram of $\geq$ is, see the figure, the graph of the relation $S = \{(n+1, n) \mid n \in N\}$. This follows from the previous item and noting $>$ is the converse of $<$. $>$ is not well-founded as from any n, there is an infinite descending chain $\ldots > n+2 > n+1 > n$.

c) The Hasse diagram of $\leq_{\text{lex}}$ on $\{a\}$ is, see the figure, the graph of the relation $S = \{(a^n, a^{n+1}) \mid n \in N\}$. This follows by noting that $a^n <_{\text{lex}} a^m$ iff $n < m$, so that the answers are 'the same' as in the first item, replacing natural numbers n by words a^n.

d) For the partial order $\leq_{\text{lex}}$ on $\{a, b\}$ with $a \leq b$, there is *no* least relation S with $S^* = \leq_{\text{lex}}$. For a proof by contradiction, suppose such an S were to exist, and consider $a <_{\text{lex}} b$. Then $a\ S^*\ b$, and we even have $a\ S^+\ b$ as $a \neq b$, so some path $a\ S \ldots S\ w\ S\ b$. Observe that w being between a and b, it must have shape av for some word v, and thus, appending a to it, also $av <_{\text{lex}} ava <_{\text{lex}} b$ and $av\ S^+\ ava\ S^+\ b$. Therefore, there is an S-path of length at least 2 from av to b. As being least entails S is irreflexive, (av, b) is not used in that path, (av, b) could be removed from S without changing the reflexive–transitive closure, contradicting S being least. Since $\ldots aab \leq_{\text{lex}} ab \leq_{\text{lex}} b$ well-foundedness does not hold.

e) For divisibility on $\mathbb{R}'$ there is no Hasse diagram, because if x divides y, there is some z such that x divides z and z divides y (we say the relation is *dense*: between any two elements there is another one). For suppose $x \cdot z' = y$ for some $z' > 1$. Setting $z = x \cdot \sqrt{z'}$, we have x divides z and z divides y since $x \cdot \sqrt{z'} \cdot \sqrt{z'} = y$ and $\sqrt{z'} > 1$. Reasoning as in the previous item this contradicts existence of a suitable *least* subrelation. Repeatedly taking the square root starting from, say, 2 shows non-well-foundedness.

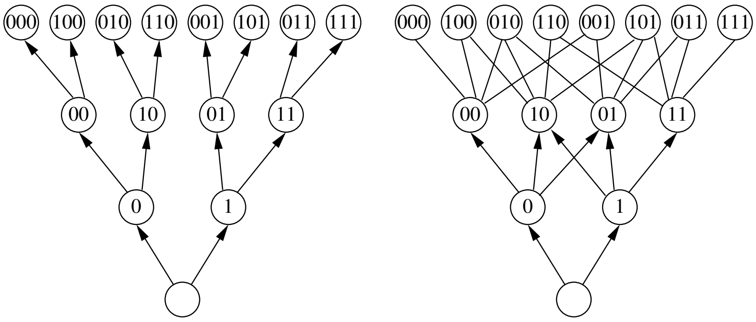
f) For divisibility on $\mathbb{N}'$ the Hasse diagram is, see the figure, the graph of the relation $S = \{(n, p \cdot n) \mid n \in N', p \text{ a prime number}\}$, by reasoning as in the first item, since prime numbers are not further decomposable. It is well-founded as there is no descending chain longer than $\log n$ from any n. The prime numbers are minimal.



2) a) see the left graph below; if $P(\epsilon)$ and if for all w, $P(w)$ entails $P(0w)$ and $P(1w)$, then for all w, $P(w)$. R_1^+ is the suffix relation;

b) see the right graph below (where arrow-heads have been omitted at the top to avoid clutter); if $P(\epsilon)$ and if for all w_1,w_2, $P(w_1w_2)$ entails $P(w_1 0 w_2)$ and $P(w_1 1 w_2)$, then for all w, $P(w)$. R_2^+ is the subsequence relation;

c) see the left graph below, and reverse all words; if $P(\epsilon)$ and if for all w, $P(w)$ entails $P(w0)$ and $P(w1)$, then for all w, $P(w)$. R_3^+ is the prefix relation.



3) Postponed

4∗) Let $\leq$ be the partial order with predecessor/strict part given by $0 < 1$. The predecessor/strict part of the componentwise extension $\leq_{\text{comp}}$ of $\leq$ relates $(0, 1)$ to $(1, 1)$ since $0 \leq 1$ and $1 \leq 1$ but $(0, 1) \neq (1, 1)$. However, the componentwise extension $<_{\text{comp}}$ of the predecessor/strict part $<$ does not relate $(0, 1)$ to $(1, 1)$ since although $0 < 1$, *not* $1 < 1$.