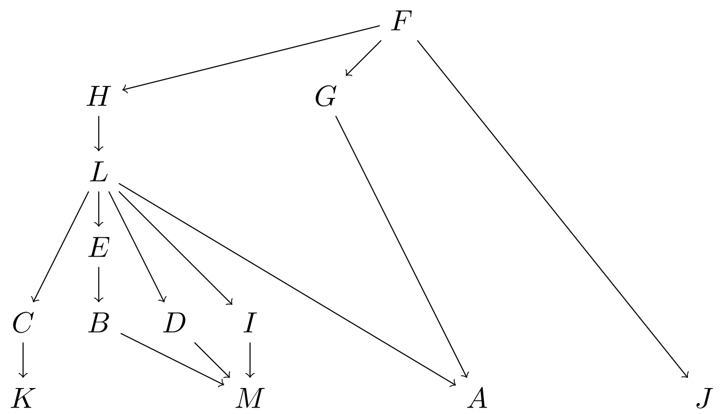
1)

a)



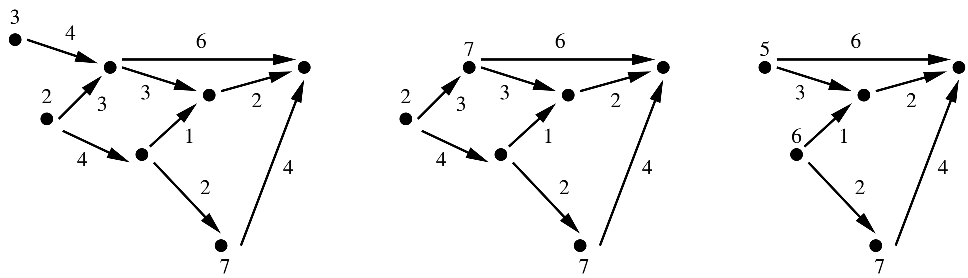
b) The source-files A, J, K and M are the only ones not imported by any source-file, and could therefore be independent programs. Hence the project comprises at most 4 of those.

c) The following topological sorting leads to a successful compilation

$$F, H, G, L, E, C, B, D, I, K, M, A, J.$$

Note that there is a lot of freedom. E.g. J could be positioned *anywhere* after F.

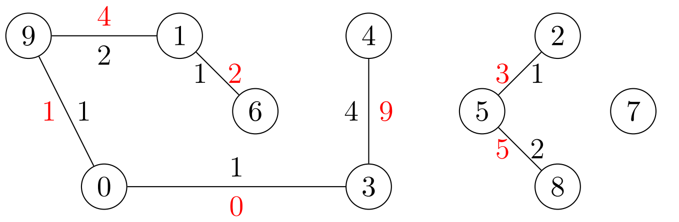
2) The first three intermediate stages after repeated removal of the leftmost node are:



Continuing this yields that the sink is reached by the path of weight 9 via the edges of respective weights 2, 4, 1, 2.

To adapt the algorithm, we update labels by each time taking the maximum instead of the minimum, yielding that the sink can be reached by the path 3, 4, 6 with weight 13, and that is maximal.

3) We remove parallel edges and loops and sort the remaining edges: $1, 3, 4, 6, 5, 11, 0, 7, 9, 8, 10, 2$. The following graph illustrates the result of applying Kruskal's algorithm, where red numbers indicate the step in the algorithm in which the edge is added to the tree, and the the black numbers indicate the edge weights.



The solution is not unique. Another spanning forest with minimal weight is obtained by using edge 8 instead of 10.

4∗) Let $\leq$ be a partial order on the finite set A. Let ℓ_0 be the empty list, and let for $n > 0$, $\ell_n = \ell_{n-1}a_n$ with a_n $\leq$-minimal in $A - \ell_{n-1}$, if there is such an a_n and otherwise $A_n = A_{n-1}$. We claim that $\forall n$, ℓ_n is a topologically $\leq$-sorted subset of A of size n, and no element of $A - \ell_n$ is $\leq$-related to an element in ℓ_n. The proof is by induction on n.

In the base case $\ell_n = ()$ and we trivially conclude as it has no elements.

In the step case $\ell_n = \ell_{n-1}a_n$ for $n > 0$, with a_n $\leq$-minimal in $A - \ell_{n-1}$. By the IH for ℓ_{n-1}, ℓ_{n-1} is topologically $\leq$-sorted and no element of $A - \ell_{n-1}$ is $\leq$-related to an element in ℓ_{n-1}. We check both properties hold for ℓ_n as well.

- To prove that ℓ_n is topologically sorted we have to show that $a_i < a_j$ entails $i < j$. Suppose $a_i < a_j$ We distinguish cases on whether i or j are n or not.

  Suppose $i, j \neq n$. Then we conclude by ℓ_{n-1} being topologically sorted by the IH.

  Suppose $i = n$ and $j \neq n$. Then $a_i \in A - \ell_{n-1}$ and $a_j \in \ell_{n-1}$, so by the IH for ℓ_{n-1}, we have that a_i is not $\leq$-related to a_j. Contradiction, so this case cannot occur.

  Suppose $i \neq n$ and $j = n$. Then $i < n$ and we conclude.

  The case $i = j = n$ cannot occur as $<$ is irreflexive as strict part of the partial order $\leq$.

- To prove that no element of $A - \ell_n$ is $\leq$-related to an element in $\ell_n = \ell_{n-1}a_n$, first observe that since $A - \ell_n = (A - \ell_{n-1}) - \{a_n\}$ we have by the IH for ℓ_{n-1} that not element of $A - \ell_n$ is $\leq$-related to an element in ℓ_{n-1}. By the choice of a_n being minimal in $A - \ell_{n-1}$, no element of $A - \ell_n$ is $\leq$-related to a_n.

We conclude by noting that $\leq$ being a finite partial order on A, any subset of A has a $\leq$-minimal element, so for n equal to the number of elements of A, we must have $A - \ell_n = \emptyset$, so ℓ_n is a topological sorting of A.