1) a) The numbering of A is the inverse $e^{-1} : A \to \{0,1,2,3\}$ of the given enumeration e, that is, $c \mapsto 0$, $a \mapsto 1$, $b \mapsto 2$, $d \mapsto 3$. From the domain of e being $\{0,1,2,3\}$, it follows the cardinality of A is $3+1=4$.

The numbering of $A\times B$ is the inverse of its enumeration, as generated by the enumerations $e : \{0,1,2,3\} \to A$ of A, $f : \{0,1\} \to B$ of B. Because the cardinalities of A and B are 4 and 2, the enumeration $A\times B$, is a function, let's call it, $(e\times f) : \{0,\ldots,4\cdot 2-1\} \to A\times B$. It is given by $k \mapsto (e(k \div 2), f(k \mod 2))$, so $0 \mapsto (e(0), f(0)) = (c,\beta)$, $1 \mapsto (e(0), f(1)) = (c,\alpha)$, $2 \mapsto (e(1), f(0)) = (a,\beta)$, $3 \mapsto (a,\alpha)$, $4 \mapsto (b,\beta)$, $5 \mapsto (b,\alpha)$, $6 \mapsto (d,\beta)$, $7 \mapsto (e(3), f(1)) = (d,\alpha)$. The corresponding numbering $(e \times f)^{-1}$ is thus given by mapping $(c,\beta),(c,\alpha),(a,\beta),(a,\alpha),(b,\beta),(b,\alpha),(d,\beta),(d,\alpha)$ to $0,\ldots,7$, respectively. The cardinality of the product $A \times B$ is the product $4\cdot 2 = 8$ of their cardinalities.

b) Since $A\cap B = \emptyset$, the enumeration of $A\cup B$, let's call it $(e\cup f) : \{0,\ldots,4+2-1\} \to A\cup B$ is given by $0 \mapsto e(0) = c$, $1 \mapsto e(1) = a$, $2 \mapsto b$ and $3 \mapsto d$ since $0,1,2,3 < 4$, and $4 \mapsto f(4-4) = f(0) = \beta$, $5 \mapsto f(1) = \alpha$ since $4,5 \not< 4$. The cardinality of the sum (disjoint union) $A \cup B$ is the sum $4+2=6$ of their cardinalities.

The enumeration of A^B, let's call it $(e^f) : \{0,\ldots,4^2-1\} \to A^B$, i.e. mapping numbers to functions $B \to A$, is given by (cf. how truth tables are usually presented):

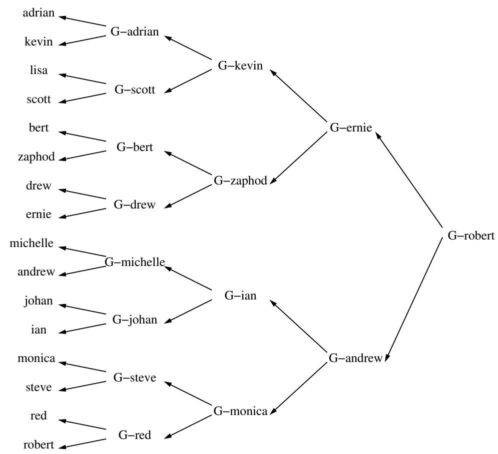
k	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
k_1k_0	00	01	02	03	10	11	12	13	20	21	22	23	30	31	32	33
$(e^f)(k)(\beta)$	c	a	b	d	c	a	b	d	c	a	b	d	c	a	b	d
$(e^f)(k)(\alpha)$	c	c	c	c	a	a	a	a	b	b	b	b	d	d	d	d

For instance, since $k=6$ is represented by $k_1k_0 = 12$ in base-4, $(e^f)(6)$ is the function that maps $f(0) = \beta$ to $e(k_0) = e(2) = b$ and $f(1) = \alpha$ to $e(k_1) = e(1) = a$, as in the table. The cardinality of the power A^B is the power $4^2 = 16$ of their cardinalities.
Note that (e^f) is *higher-order* in the sense that for any input (the number k) its output is a *function* (from B to A).

c) $\#(A\cup B\cup C) = (\#A + \#B + \#C + \#(A\cap B\cap C)) - (\#(A\cap B) + \#(B\cap C) + \#(C\cap A)) = (4+2+3+0) - (0+2+1) = 9-3 = 6$. Indeed, $A\cup B\cup C = \{a,b,c,d,\alpha,\beta\}$.

2) a) Since $\frac{17}{5} > 3$ the maximum number of pigeons in a hole is greater than 3, i.e. is at least 4, using the lemma on slide 17 of lecture 7.

b)
- Choosing Lisa as winner of the tournament, and Robert as winner of the other semi-final, and naming game-nodes after the player that *loses* that game, we have:



(Note that taking as non-leaf-nodes, i.e. games, a node named after the player that *wins*, does not yield a tree but a (cyclic) graph, since players may win several games.)

- The bijection is already given by the naming of game-nodes, mapping game-node G-x to player x. There are as many games as there are players that lose, i.e. all players minus one. There are 16 players, so 15 games.

The reasoning in the previous item is general.
Note: we may also conclude from this that the number of leafs in a binary tree is its number of non-leaf-nodes plus one.

3) a) each element of A can be mapped to each element of B, so 10^6; after swapping 6^{10};
   b) the falling factorial $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 151200$; after swapping 0;
   c) 0 as A and B have different cardinalities; only this answer remains the same;
   d) $2^6 = 64$; after swapping $2^{10} = 1024$;
   e) $\binom{6}{7} = 0$; after swapping $\binom{10}{7} = \frac{10\cdot 9\cdot 8}{3\cdot 2\cdot 1} = 120$.

4∗)
- If $n \leq m$, then $\max(n,m) = m = (n+m) - n = (n+m) - \min(n,m)$. The case $m \leq n$ is symmetrical.
- By exactly the same proof as on slide 12 of lecture 7, replacing (cardinalities of) intersections and unions with minimum and maximum.

5∗)
- $2^{\#D \cdot \#D} = 2^{n \cdot n} = 2^{n^2}$ resp. $2^0 = 1$, 2^n, 2^{n^3}.
- 2^n resp. $2^0 = 1$, $2^{\#D \cdot \#D} = 2^{n \cdot n}$, 2^{n^3}.

That is, there are exactly the same number of n-ary relations on D, i.e. *subsets* of D^n, as there are n-ary predicates on D, i.e. *characteristic* functions from D^n to $\{F, T\}$. Predicates can be thought of both as subsets and as characteristic functions.