universität innsbruck

Lechner Group

FWF Der Wissenschaftsfonds.
START 2017

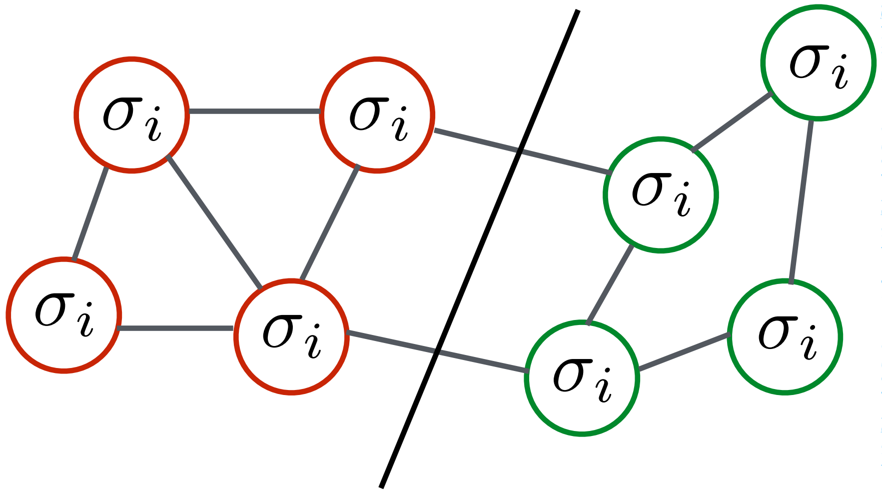
Enhancing adiabatic quantum computation with non-adiabatic methods

Andreas Hartmann
Quantum optimization group
11.12.2019

Lechner Group

Motivation

Graph Partitioning

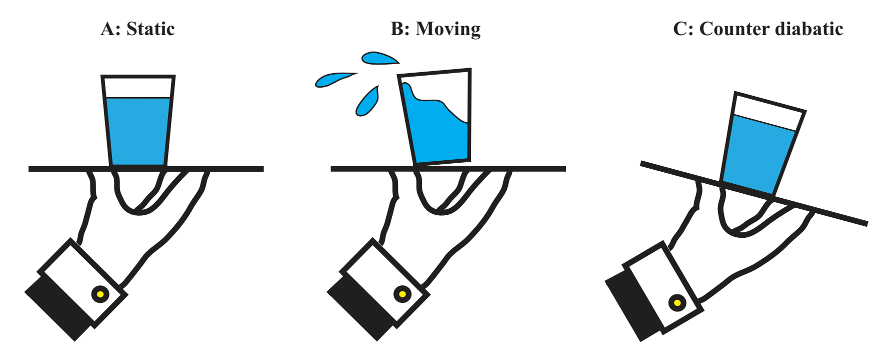


Adiabatic Quantum Computation

Solving optimization problems

wikipedia.org

Non-adiabatic Driving

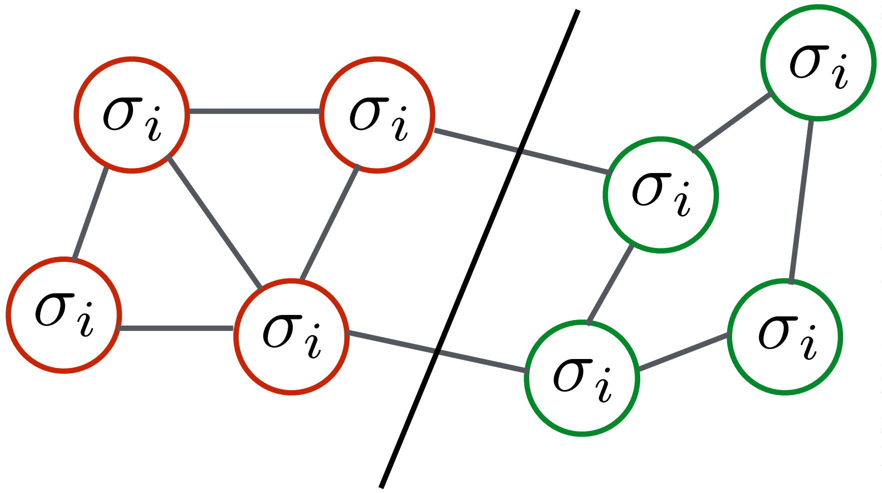


D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Possible speedup?

Motivation

Graph Partitioning



Adiabatic Quantum Computation

Solving optimization problems

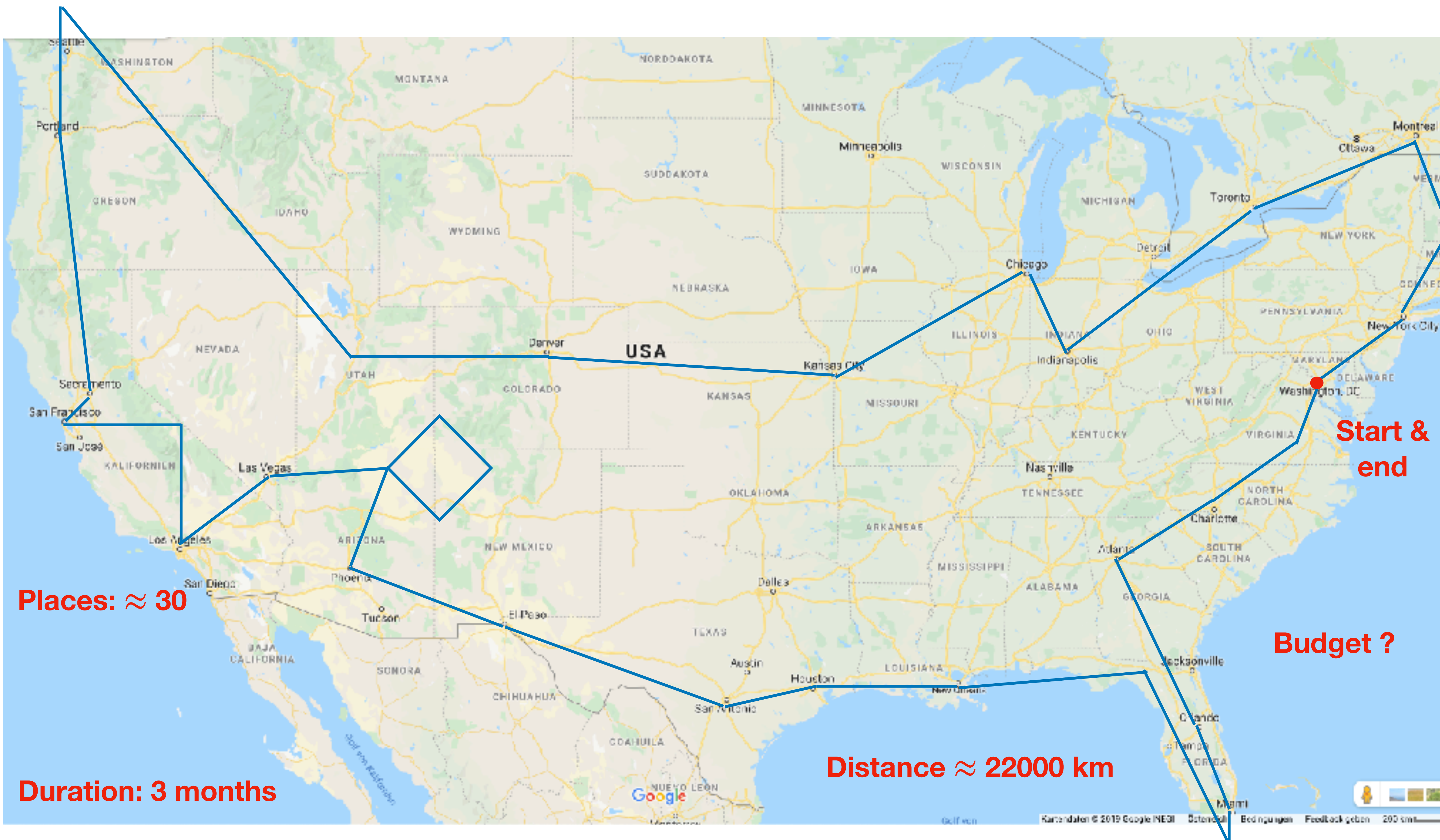
wikipedia.org

Non-adiabatic Driving

D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Possible speedup?

Traveling salesman problem



Places: ≈ 30

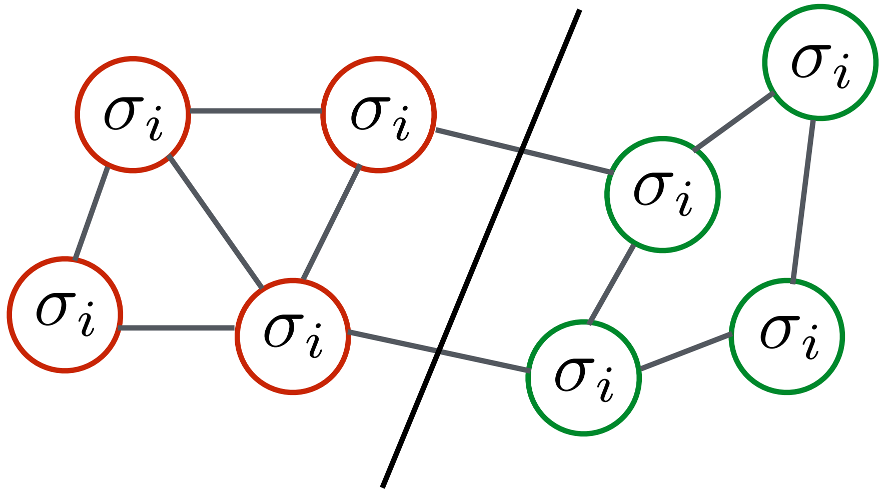
Duration: 3 months

Distance ≈ 22000 km

Budget ?

Start & end

Graph Partitioning



$G = (V, E)$

We ask: What is a partition of the set V into two subsets of equal size $N/2$ such that the number of edges connecting the two subsets is minimized?

Strategy: Cost-function = Energy

$\sigma_i = +1$ (red circle)

$\sigma_i = -1$ (green circle)

$$H_p = C_A \left(\sum_{n=1}^{N} \sigma_i \right)^2 + C_B \sum_{\{i,j\} \in E} \frac{1 - \sigma_i \sigma_j}{2}$$

Two subsets have the same size.

Connection between two sets is minimal.

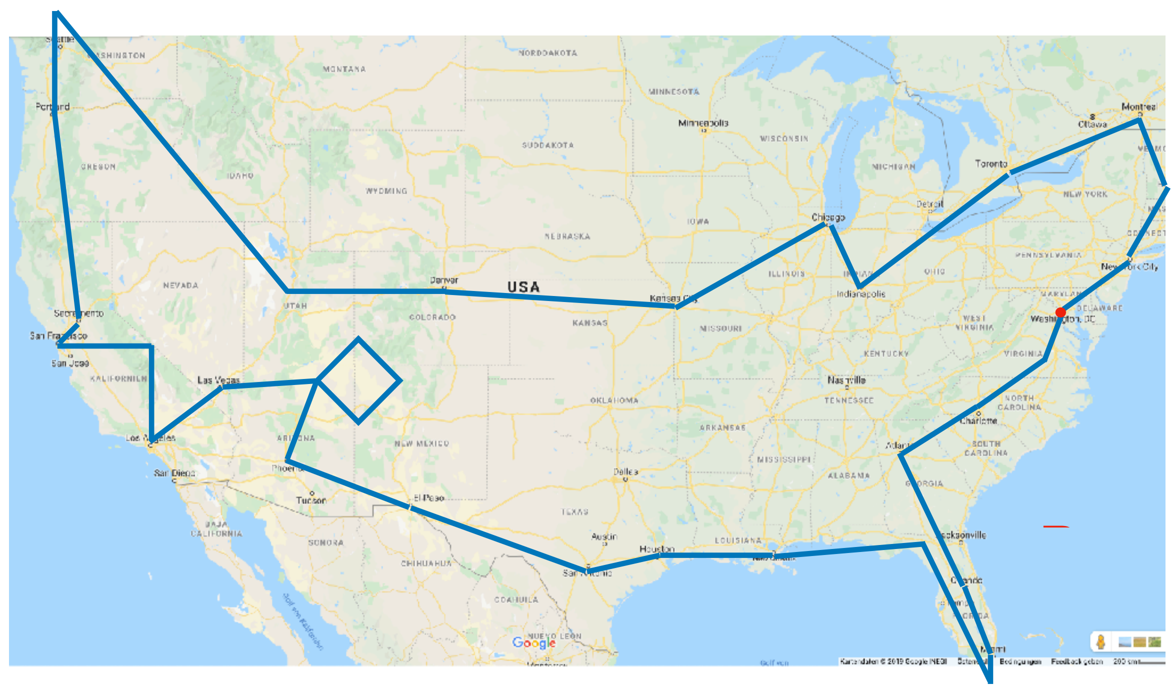
A. Lucas, Frontiers in Physics ***2****, 00005 (2014)*

Finding **ground** state = **Minimizing** costs = **optimal solution** to problem

Problem:

NP-hard

Non-deterministic **P**olynomial time



30 places

↓

N places

➡ <u>**Exponentially increasing**</u> computation time

➡ **No <u>classical</u>** algorithm can solve these problems efficiently

➡ Possible solution: Adiabatic <u>**quantum**</u> computation?

Motivation

Graph Partitioning

Adiabatic Quantum Computation

Solving optimization problems

wikipedia.org

Non-adiabatic Driving

D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Possible speedup?

Possible solution: Adiabatic **quantum** computing?

Classical computer

Basic computational unit: **1 bit**

1 0

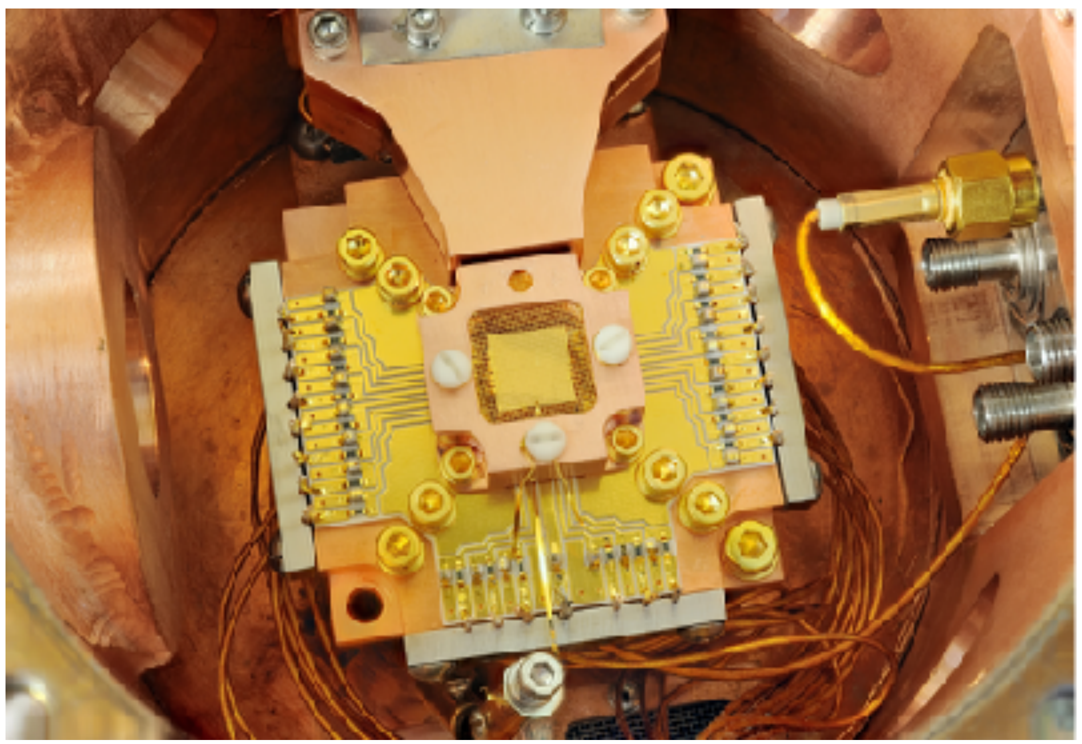
8 bits = 1 byte

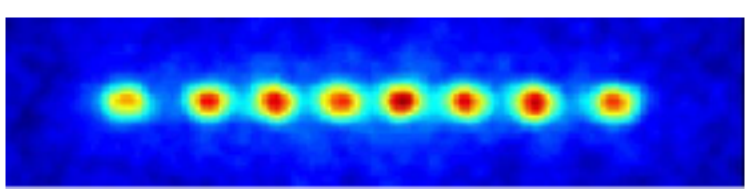
A	01000001
N	01001110
D	01000100
R	01010010
E	01000101
A	01000001
S	01010011

"Zuse 3", br.de

Possible solution: Adiabatic **quantum** computing?

Experiments:





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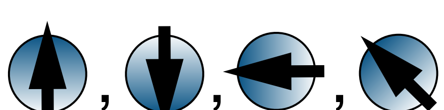
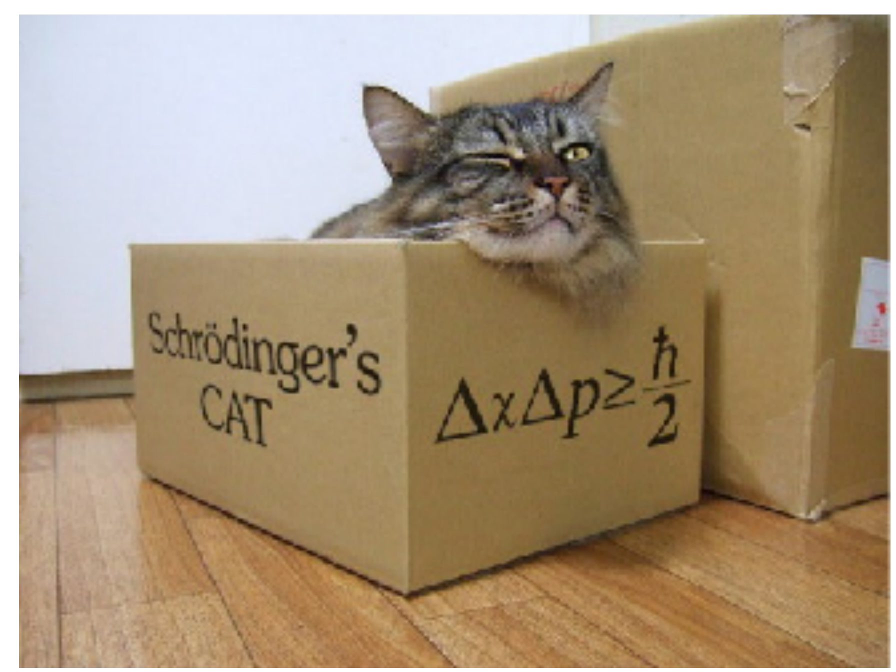
<u>Qubit:</u> spin 

<u>Quantum computer</u>

Quantum bit = **qubit**

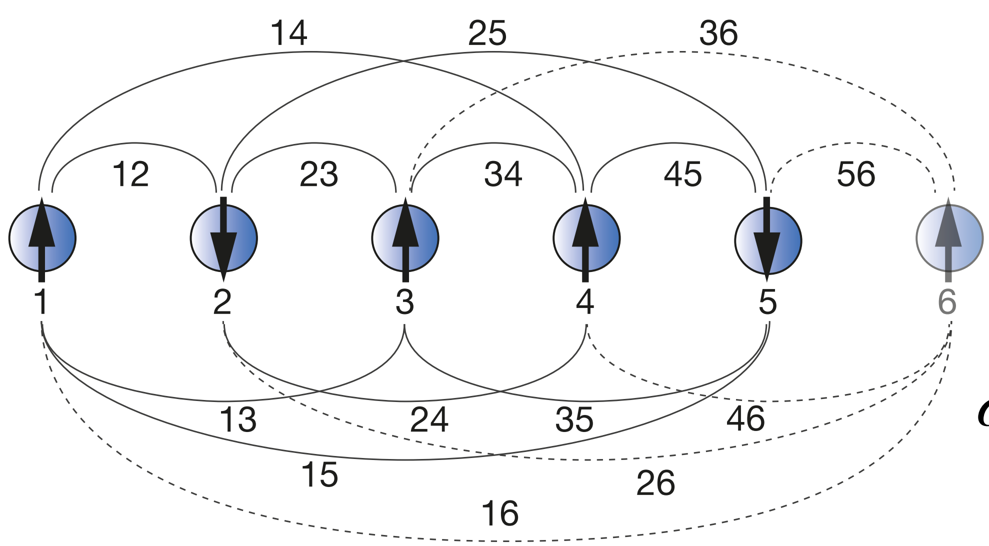
$$|\psi\rangle = \frac{|1\rangle + |0\rangle}{\sqrt{2}}$$

Superposition of two states



burks.de

Quantum annealing



Ising spin chain:

$$H(t) = A(t)\boxed{\sum_{i}^{N} b_i \sigma_i^x} + B(t)\boxed{\left[\sum_{i}^{N} h_i \sigma_i^z + \sum_{i}^{N}\sum_{j}^{i} J_{ij}\sigma_i^z \sigma_j^z\right]}$$

$$\sigma_i = \begin{cases} +1 \\ -1 \end{cases}$$

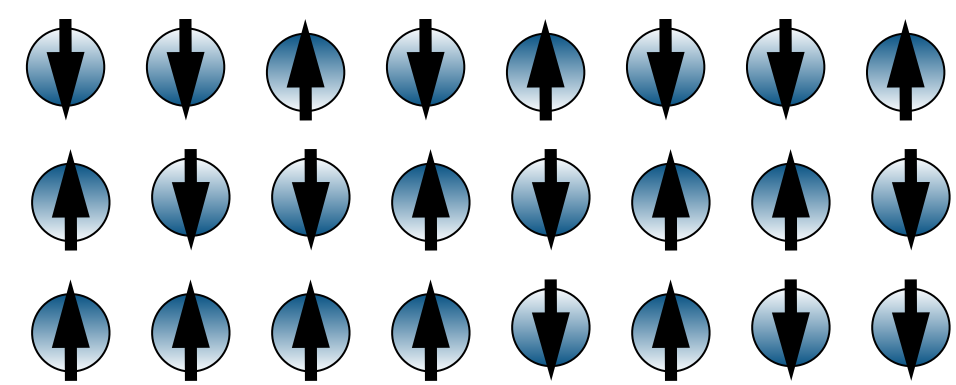
$H_D(t)$ "Driver term"

$H_P(t)$ "Problem Hamiltonian"

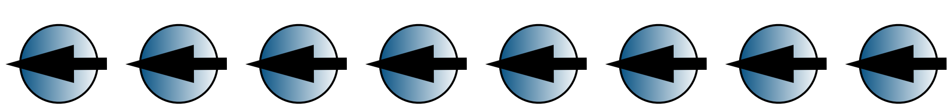
Evolution Hamiltonian: $H(t) = (1 - t/T)H_D + (t/T)H_P \qquad (0 \leq t \leq T)$

Excited states:

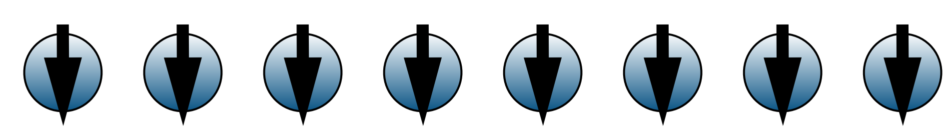
1)
2)
3)



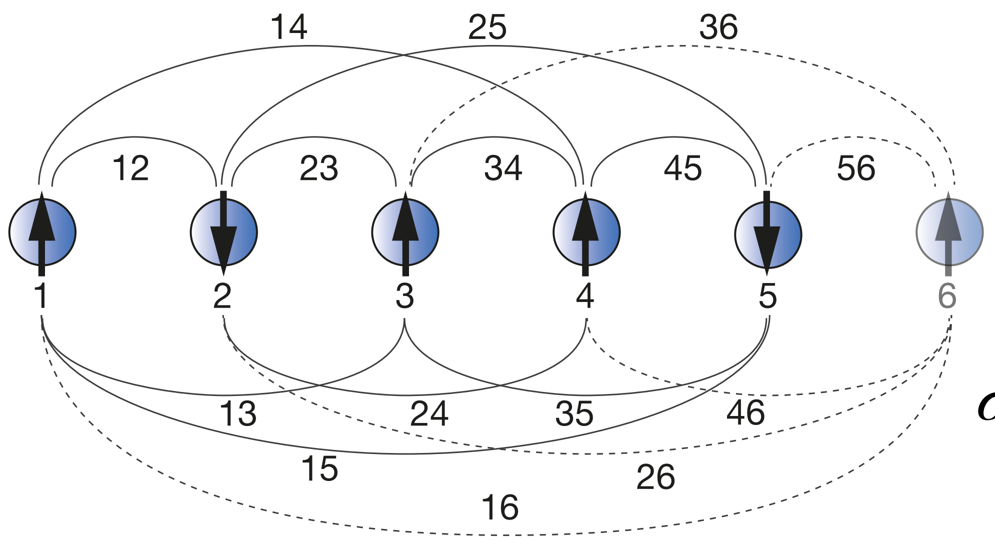
Initial state:



Ground state:



Quantum annealing



Ising spin chain:

$$H(t) = A(t)\boxed{\sum_i^N b_i\sigma_i^x} + B(t)\boxed{\left[\sum_i^N h_i\sigma_i^z + \sum_i^N\sum_j^i J_{ij}\sigma_i^z\sigma_j^z\right]}$$

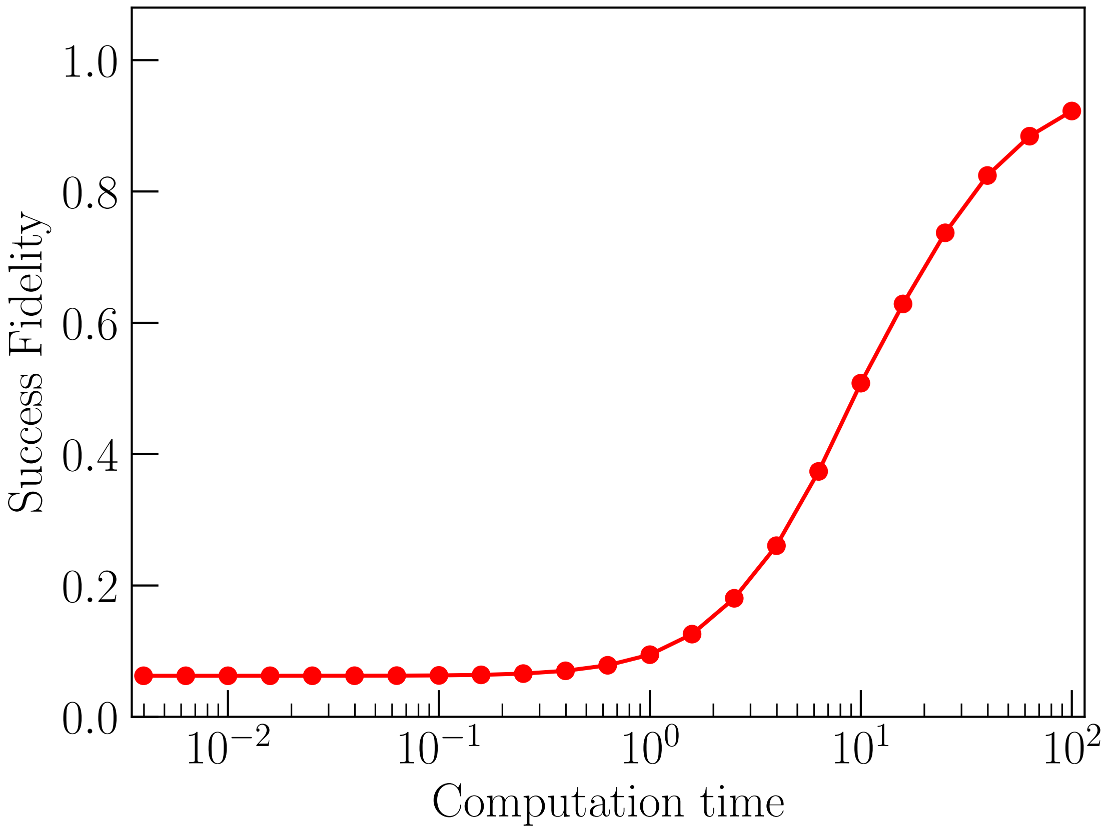
$$\sigma_i = \begin{cases} +1 \\ -1 \end{cases}$$

$H_D(t)$ "Driver term"

$H_P(t)$ "Problem Hamiltonian"

Evolution Hamiltonian: $H(t) = (1 - t/T)H_D + (t/T)H_P \qquad (0 < t < T)$

Success fidelity:

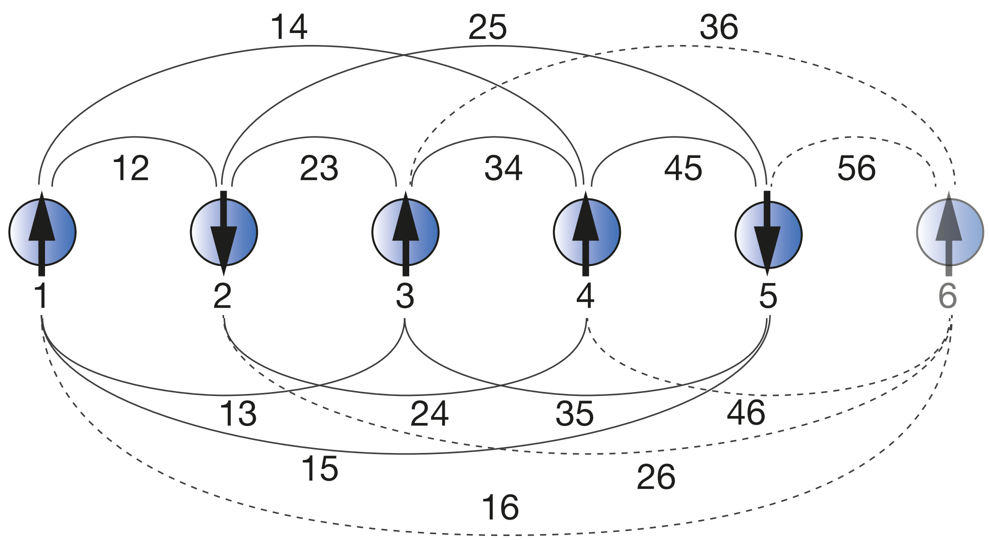


Example:

N=8 logical qubits

Problem 1: Long sweep times to find ground state

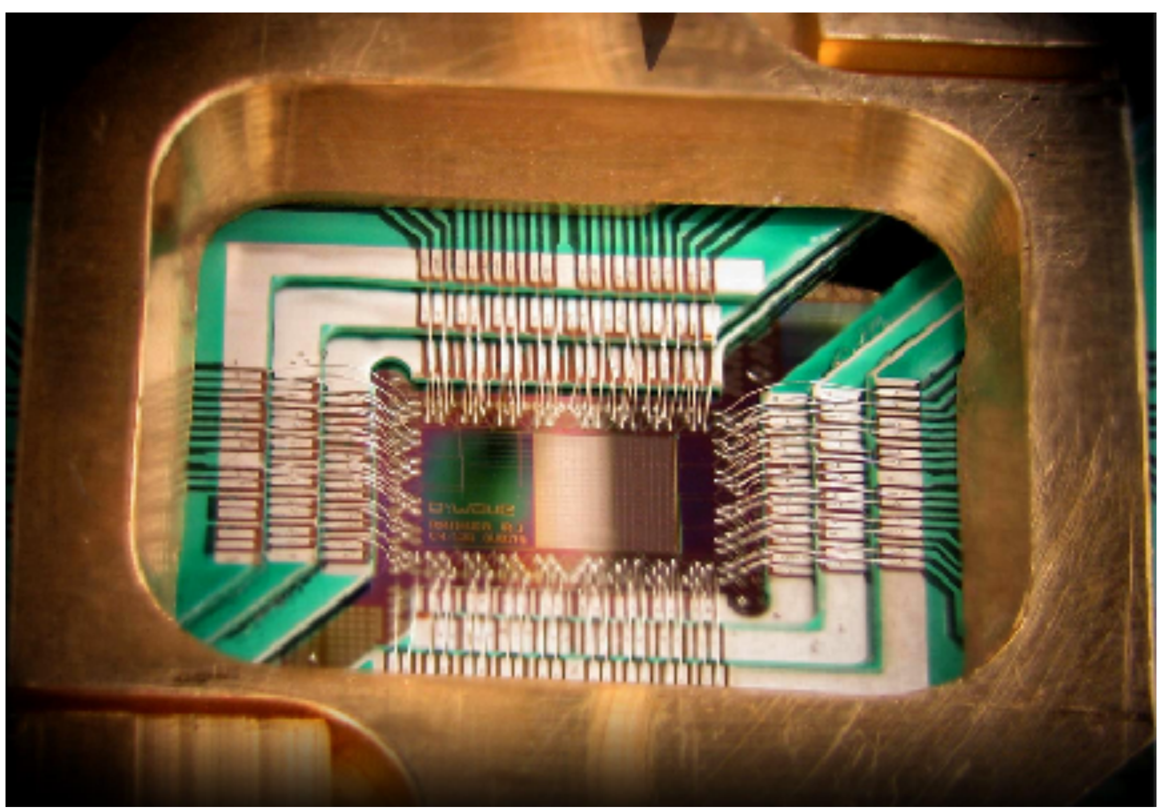
Quantum annealing



Ising spin chain:

$$H(t) = A(t)\sum_{i}^{N} b_i\sigma_i^x + B(t)\left[\sum_{i}^{N} h_i\sigma_i^z + \sum_{i}^{N}\sum_{j}^{i} J_{ij}\sigma_i^z\sigma_j^z\right]$$

DWave

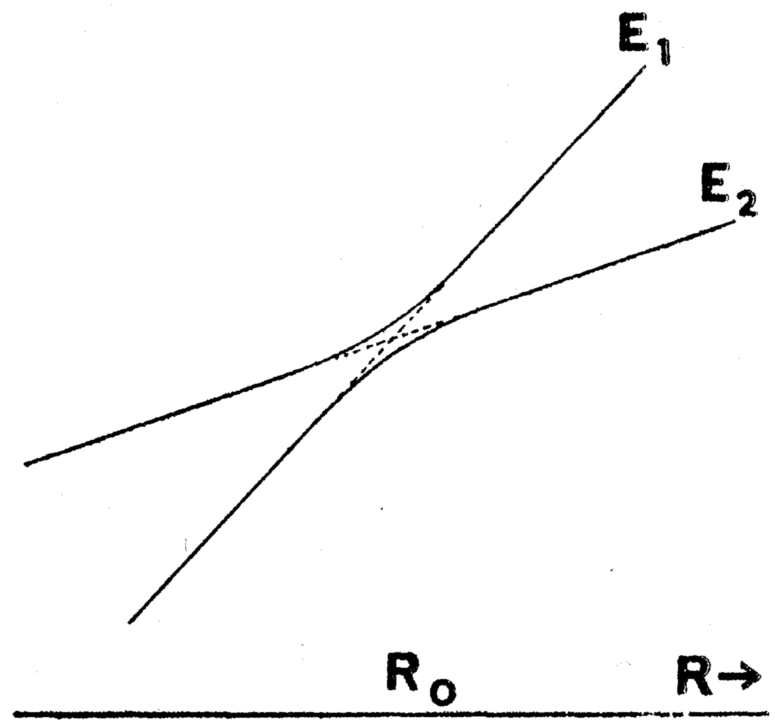


DWave

Transitionless quantum driving

Adiabatic Theorem

"A physical system remains in its instantaneous eigenstate if a given ***perturbation*** *is* ***acting on it slowly enough*** *and if there is a* ***gap*** *between the eigenvalue and the rest of the Hamiltonian's spectrum." Adiabatic theorem (Born and Fock, 1928).*



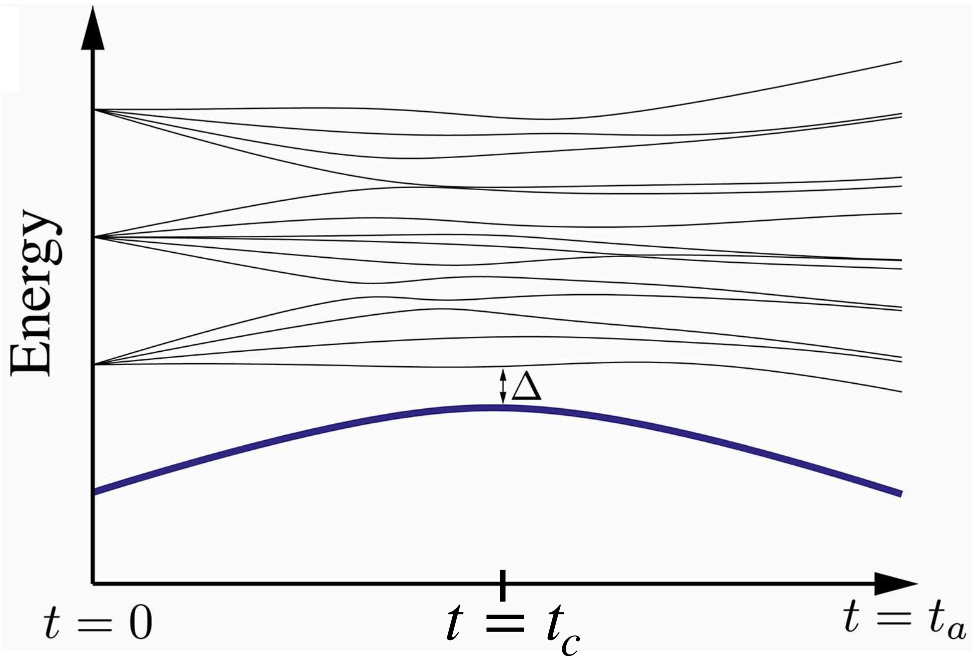
Zener, C, (1932)

Time to solution: $T \propto \frac{|\langle 1|dH/dt|0\rangle|}{\Delta^2}$

Scaling:

1st order QPT: $\Delta \propto e^{-aN}$ ➡ $T \propto e^{aN}$

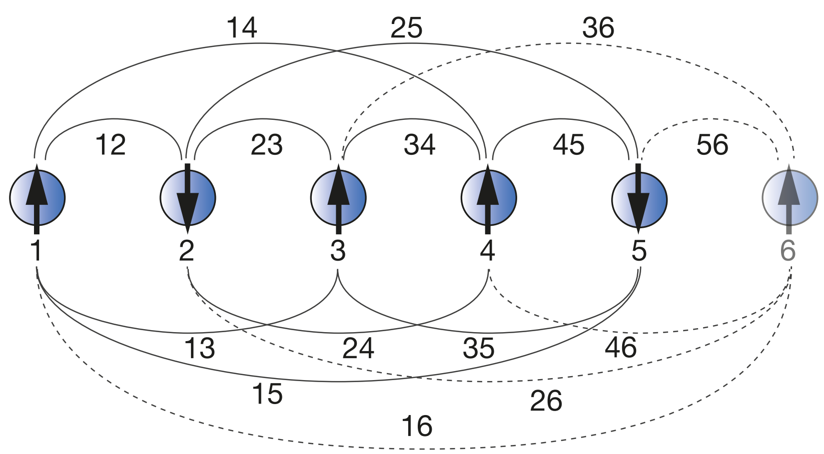
2nd order QPT: $\Delta \propto N^{-\alpha}$ ➡ $T \propto N^{\alpha}$



Problem 2: **Exponentially** closing minimal gap/increasing computation time!

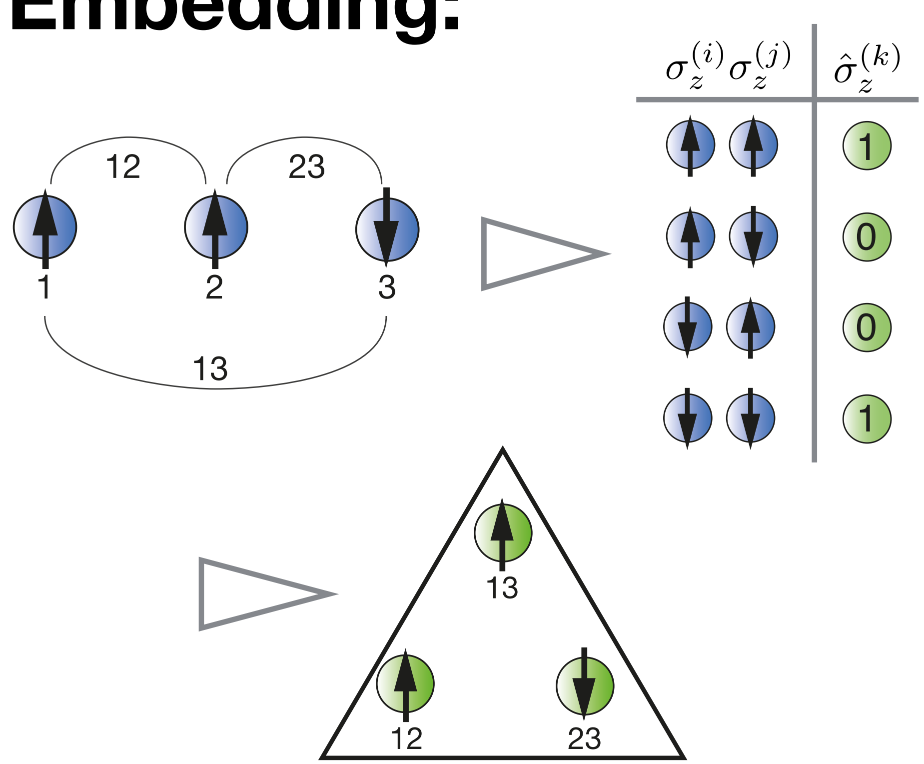
AH and W. Lechner, PRA **100** 03205 (2019)

Spin glass paradigm



$$H(t) = A(t)\sum_i^N b_i\sigma_i^x + B(t)\left[\sum_i^N h_i\sigma_i^z + \sum_i^N\sum_j^i J_{ij}\sigma_i^z\sigma_j^z\right]$$

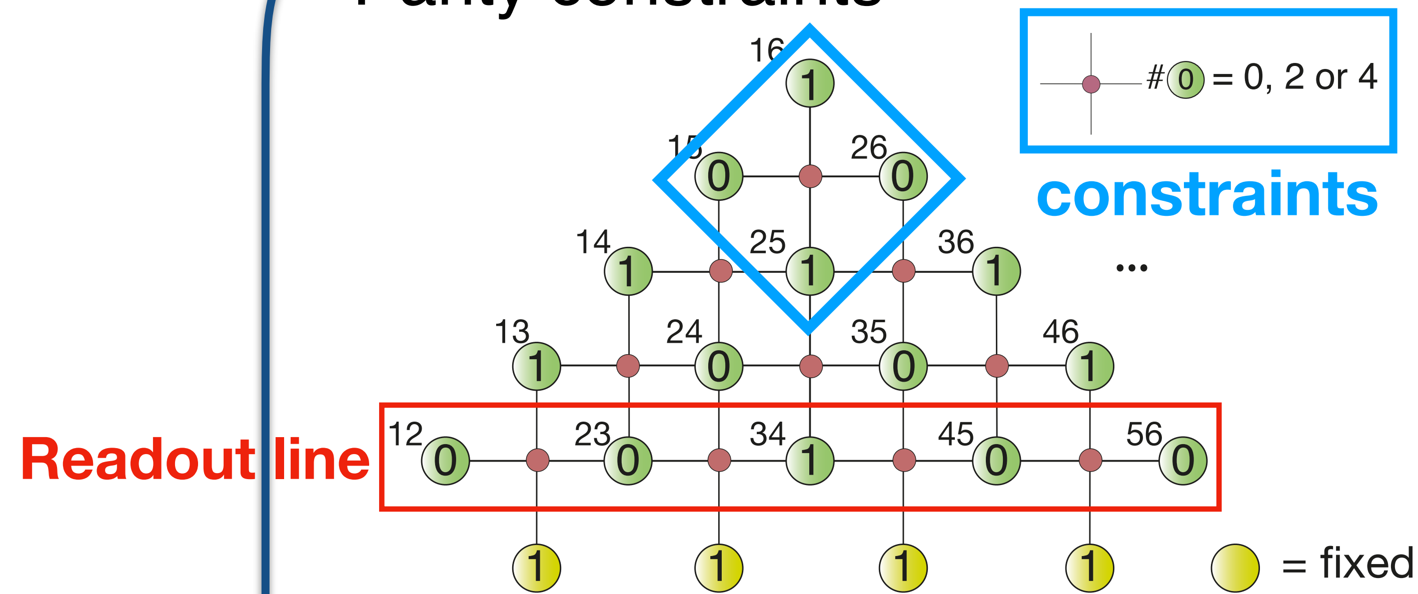
Embedding:



Lattice gauge model (LHZ)

- Physical qubits available in the lab
- Constraints problem-independent

Parity constraints



$$H(t) = A(t)\sum_i^K b_i\sigma_i^x + B(t)\sum_i^K J_i\sigma_i^z + \sum_l^{K-N+1} C_l\sigma_{l,n}^z\sigma_{l,e}^z\sigma_{l,s}^z\sigma_{l,w}^z$$

14 W. Lechner, P. Hauke, P. Zoller, Science Advances 1, 1500838 (2015)

Quantum annealing

Speed limit: Adiabatic theorem

Problem 1
"Long sweep times"

Problem 2
"Exponentially closing gap"

Counter-diabatic driving

M. Demirplak and S.A. Rice
J.Phys.Chem.A **2003**, 107

AH and Wolfgang Lechner
New. J. Phys. **21** 043025 (2019)

Inhomogeneous Driving

Yuki Susa et al *PRA* **98** 042326 (2018)

AH and Wolfgang Lechner
PRA **100** 032110 (2019)

Future Goal: efficient ***non-adiabatic*** quantum computing

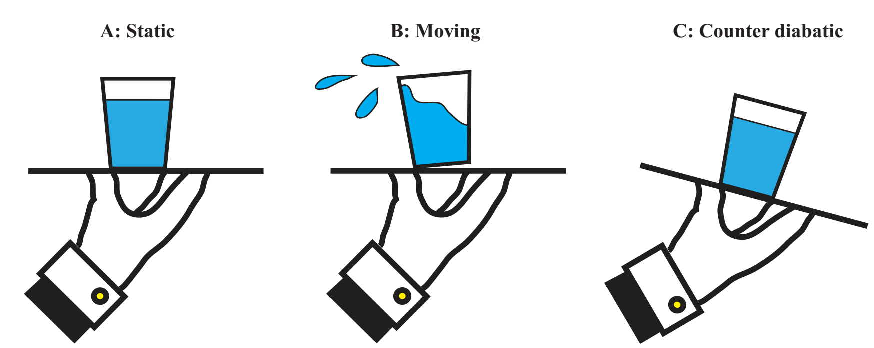
Motivation

Graph Partitioning

Adiabatic Quantum Computation

Solving optimization problems

Non-adiabatic Driving



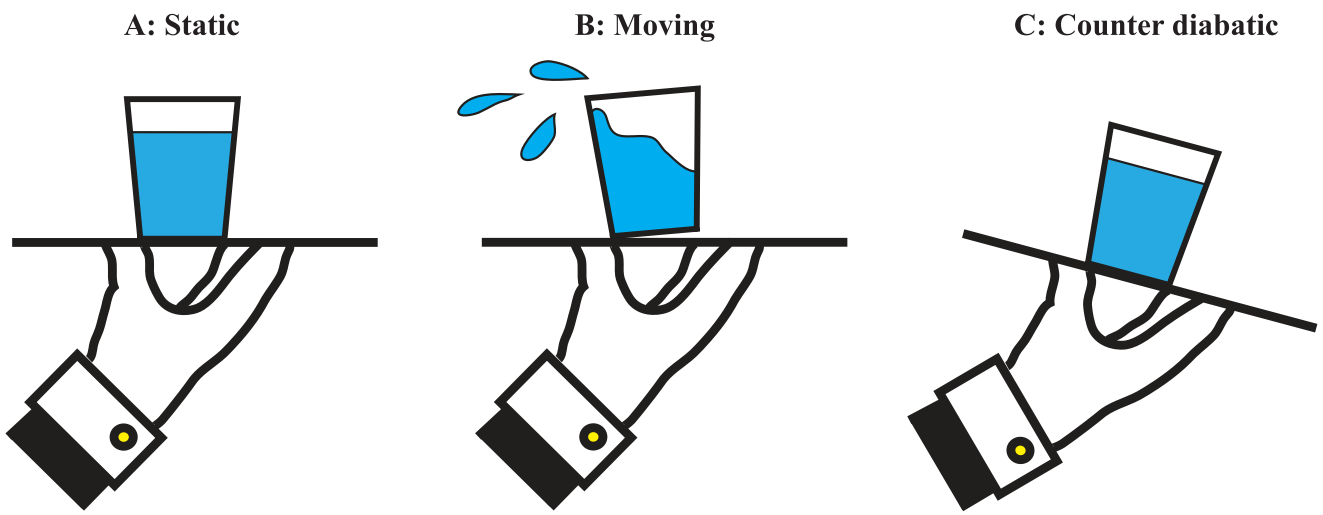
D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Possible speedup?

3. Non-diabatic Driving

Lechner Group

Basic idea



D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Goals

- Fast protocols for quantum annealing
- High ground state fidelity

Analysis:

Moving frame: $|\tilde{\psi}\rangle = U^{\dagger}(\lambda)\,|\psi\rangle$ $\xrightarrow{\text{Schrödinger Eq.}}$

Hamiltonian in moving frame:

$$\tilde{H}_m = \tilde{H} - \dot{\lambda}\tilde{A}_\lambda$$

$\tilde{H}$: **diagonal Hamiltonian**

$\tilde{A}_\lambda$: **adiabatic gauge potential responsible for transitions**

Counter-diabatic Hamiltonian: $H(t) = H_0(t) + \dot{\lambda}A_\lambda$

$$H(t) = \underbrace{\sum_n |n\rangle E_n \langle n|}_{H_0(t)} + \underbrace{i\hbar \sum_{m\neq n}\sum_n \frac{|m\rangle\langle m|\partial_t H_0|n\rangle\langle n|}{E_m - E_n}}_{H_{CD}(t)}$$

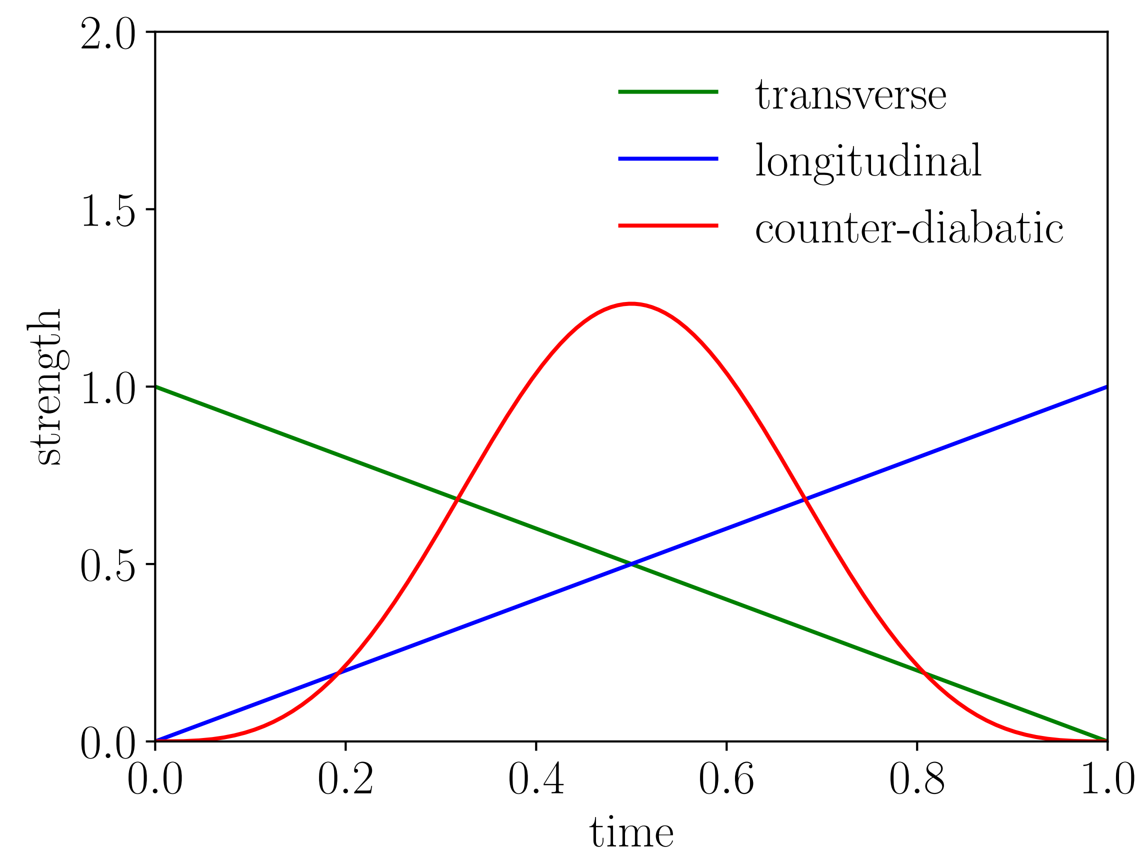
Problem: A priori knowledge of the system's eigenstates

Approximate counter-diabatic driving

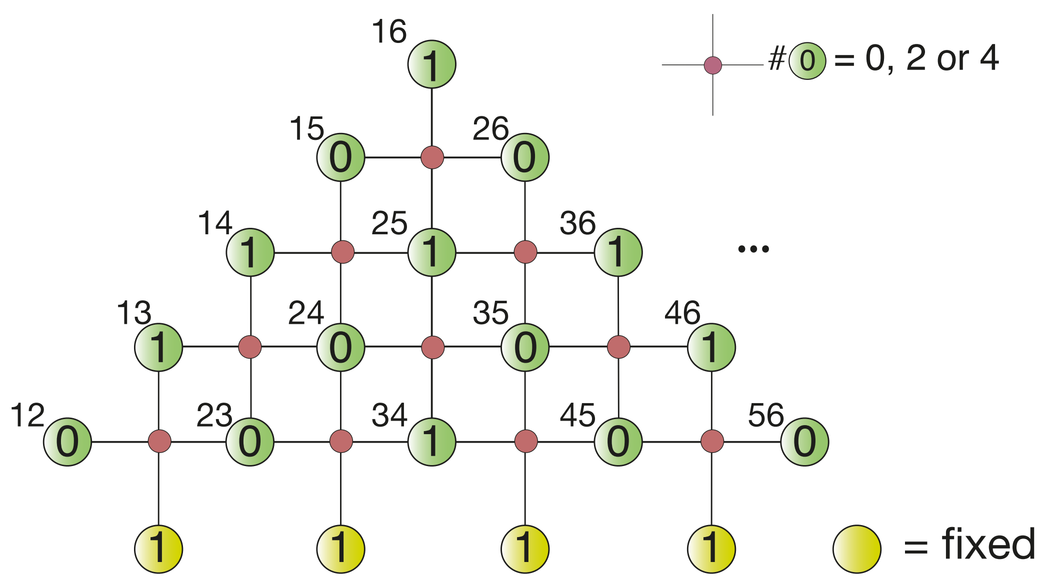
D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Looking for **approximate** solution for $\mathcal{A}_\lambda$

Make **ansatz** which we have accessible in the lab



LHZ: $$H_0(t) = \sum_{k=1}^{N_p} h_k \sigma_k^x + \sum_{k=1}^{N_p} J_k \sigma_k^z - \sum_{l=1}^{N_c} C_l \sigma_{l,n}^z \sigma_{l,e}^z \sigma_{l,s}^z \sigma_{l,w}^z$$



Ansatz: $$\mathcal{A}_\lambda^* = \sum_{i=1}^{N_p} \alpha_i \sigma_i^y$$

find optimal expression

<u>Analytical</u> variational optimization

3. Non-diabatic Driving

Problem: Just works well for ordered quantum systems

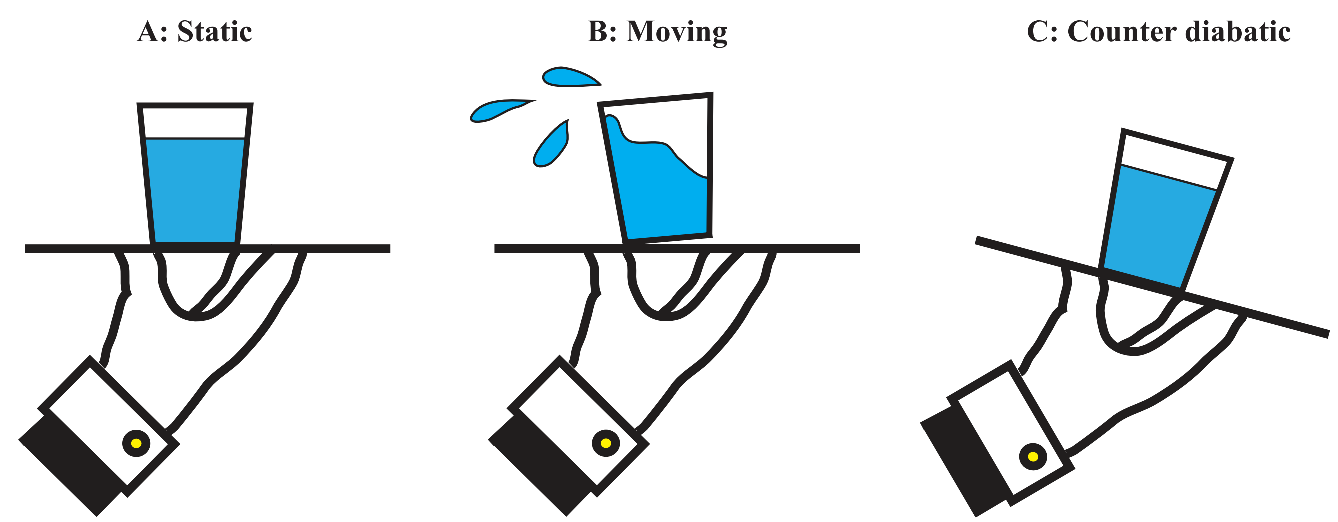
Solution: Hybrid quantum-classical iterative variation

AH and W. Lechner, 2019, *New. J. Phys.* **21** 043025

$$H(t) = H_0(t) + \dot{\lambda} A_\lambda$$

analytical variation

iterative variation



D. Sels, A. Polkovnikov, PNAS, **114** 3909 (2017)

Quantum annealing + <u>analytical</u> variation

Additional <u>iterative</u> variation of control parameter

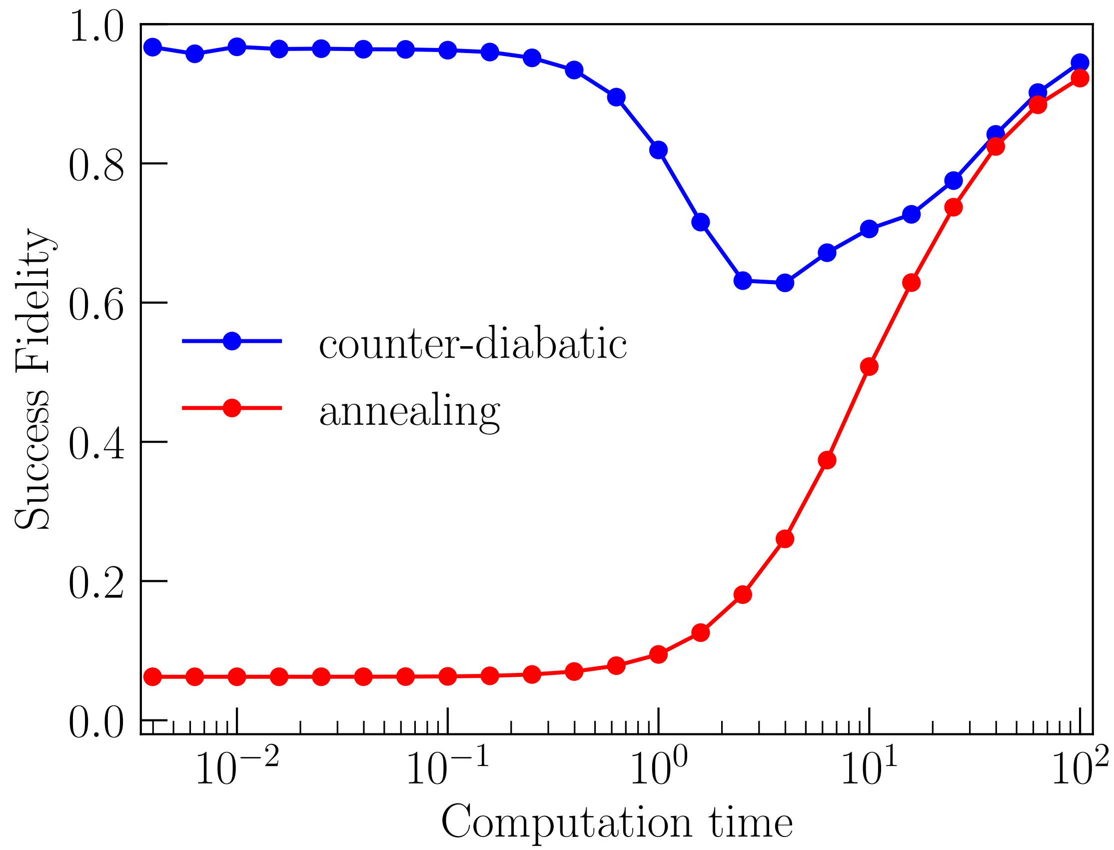
AH and W. Lechner, 2019, *New. J. Phys.* **21** 043025

Numerical Results:

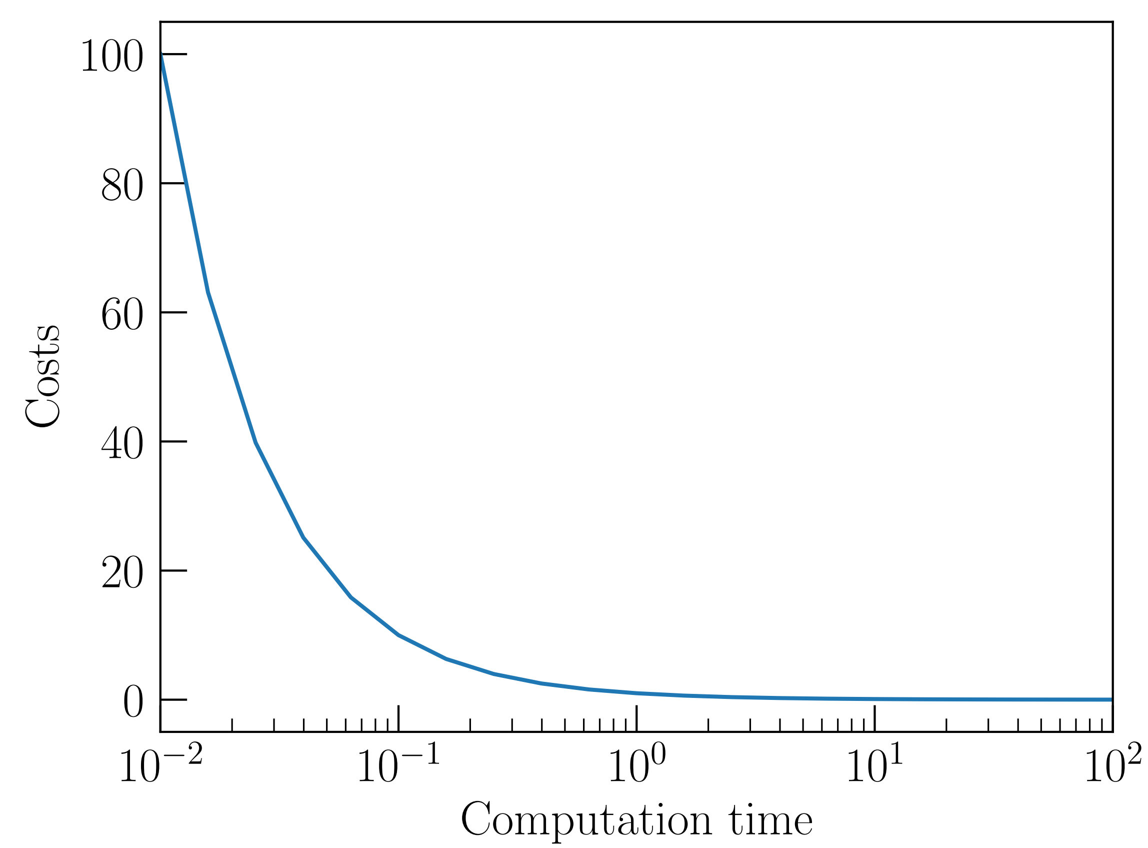
$$H_{CD,LHZ}(t) = \underbrace{\sum_{k=1}^{N_p} h_k(t)\sigma_k^x + \sum_{k=1}^{N_p} J_k(t)\sigma_k^z - \sum_{l=1}^{N_c} C_l(t)\sigma_{l,n}^z\sigma_{l,w}^z\sigma_{l,s}^z\sigma_{l,e}^z}_{H_0(t)} + \underbrace{\sum_{k=1}^{N_p} Y_k(\lambda_f, t)\sigma_k^y}_{H_{CD}(t)}$$

original Hamiltonian **additional counter-diabatic term**

Success Fidelity:



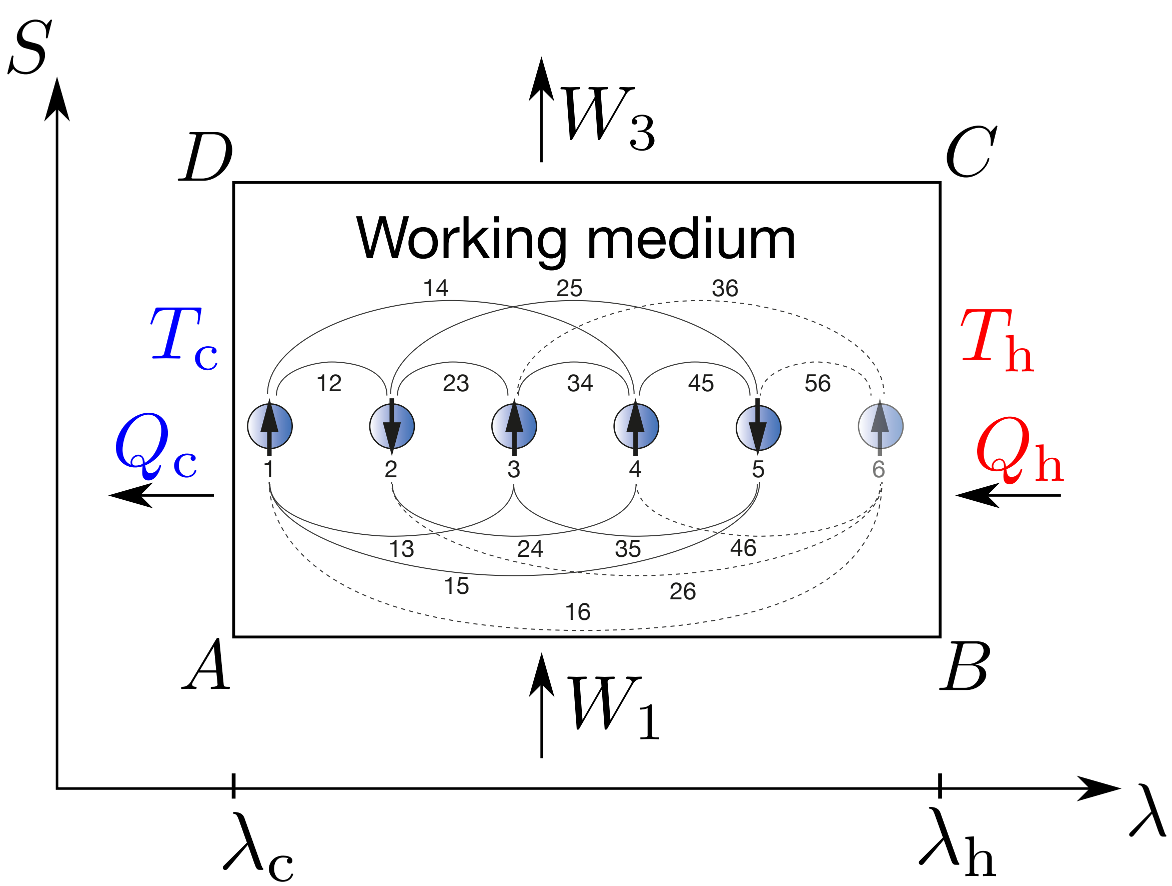
Costs: "No free lunch"

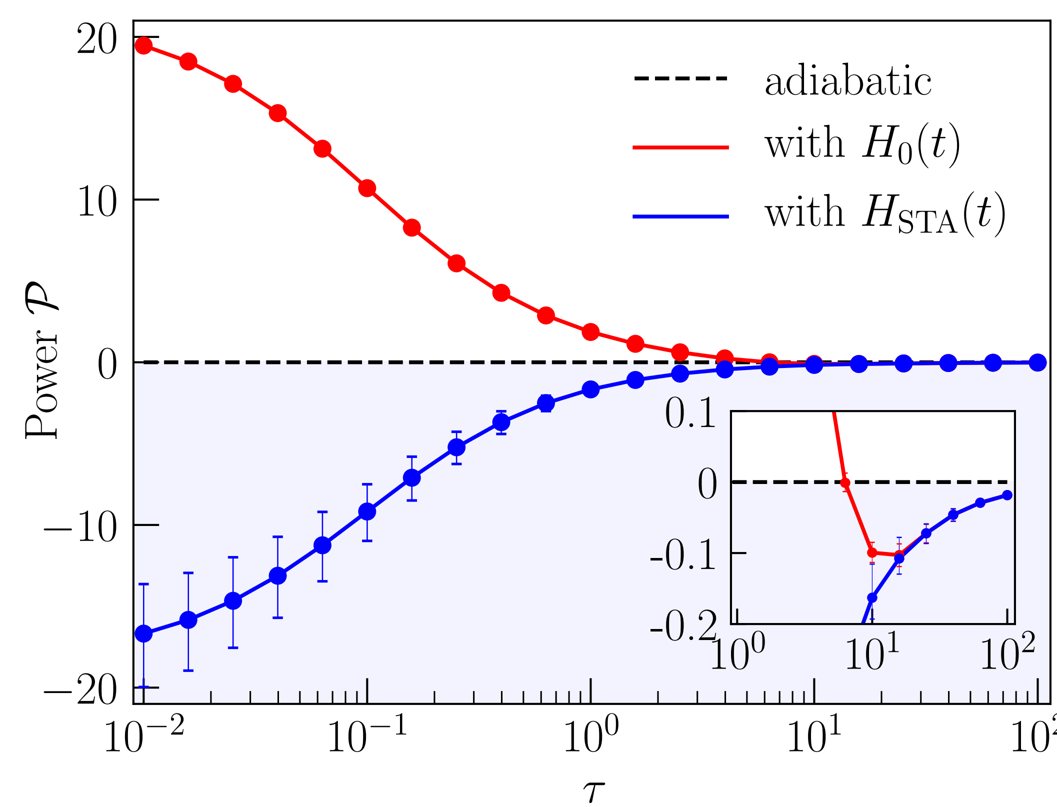


Conclusion:

We have

- … further developed a method to improve quantum annealing by counter-diabatic driving and inhomogeneous driving for lattice gauge quantum computing
- … applied this method to the field of quantum thermodynamics
- … filed a patent with this idea (applicable to all Hamiltonians)



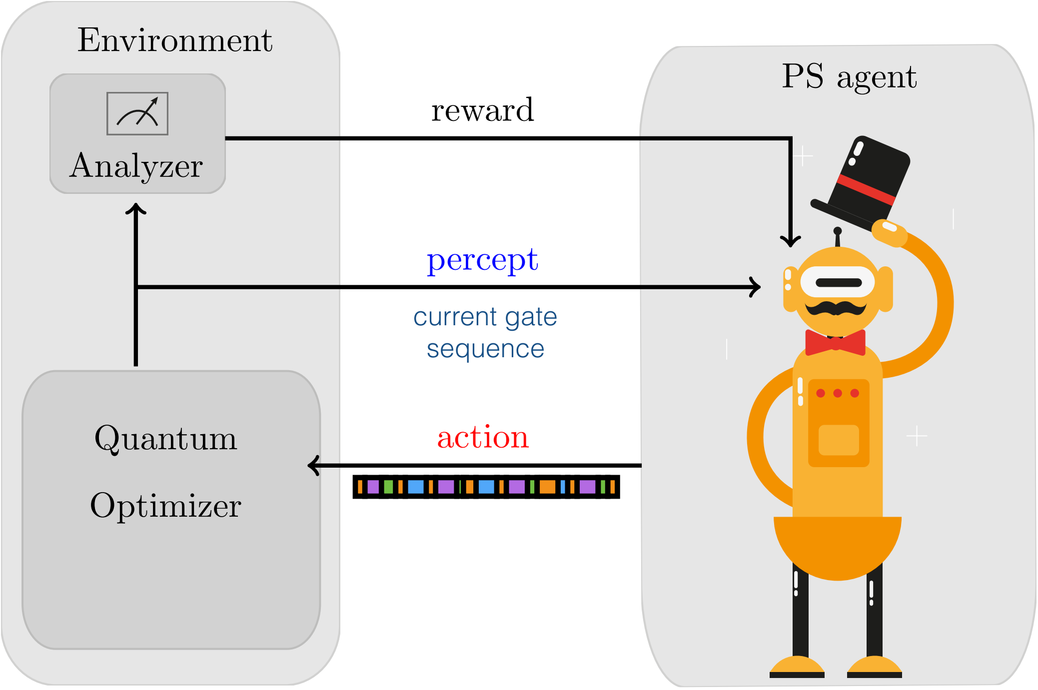


AH, V. Mukherjee, W. Niedenzu and W. Lechner, to be published

Outlook:

We want to

- … implement counter-diabatic driving to **open** quantum systems
- … implement this idea in an **experiment** (ongoing collaboration with group of Rainer Blatt)
- … implement the counter-diabatic for **artificial intelligence** (ongoing work with group of Hans Briegel)



My References:

[1] AH and W. Lechner, 2019, *New. J. Phys.* **21** 043025

[2] AH and W. Lechner, PRA **100** 03205 (2019)

[3] AH, V. Mukherjee, W. Niedenzu, and W. Lechner, *“Many body quantum heat engines with Shortcuts to adiabaticity”*, to be published

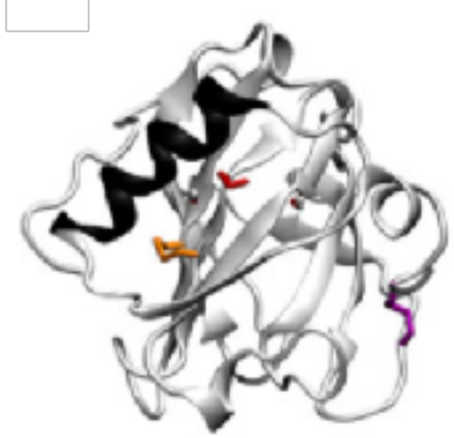
My Patents:

[1] AH and W. Lechner, 2019, “*Method of computing a solution to a computational problem using a quantum system and apparatus for computing solutions to computational problems*”, under review

Appendix

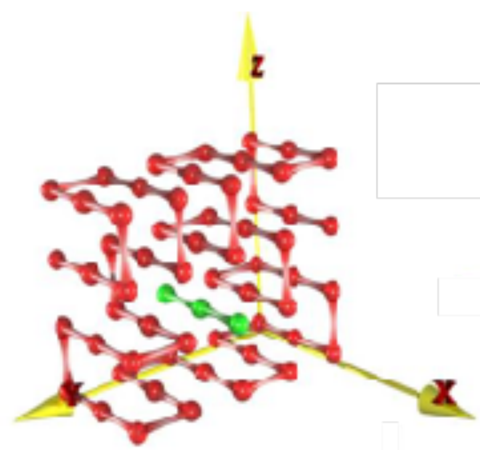
Other scientific optimization problems:

Protein folding



e.g. beta-lactoglobolin (milk protein)
Picture: Peter Bolhuis

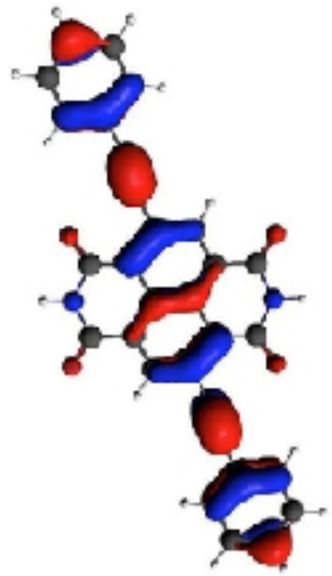
→



I. Coluzza, et.al. Biophys. J. (2007).

Adiabatic Quantum computing: A. Perdomo-Ortiz et. al., Sci. Rep. 2, 571 (2012).

Quantum Chemistry



Picture: E. Meijer, University of Amsterdam.

Adiabatic Quantum Computing: R. Babbush et. al., Sci. Rep. 4, 6603 (2014).

Quantum annealing

Speed limit: Adiabatic theorem

Problem 1
“Long sweep times”

Counter-diabatic driving
M. Demirplak and S.A. Rice
J.Phys.Chem.A **2003**, 107

AH and Wolfgang Lechner
New. J. Phys. **21** 043025 (2019)

Problem 2
“Exponentially closing gap”

Inhomogeneous Driving
Yuki Susa et al *PRA* **98** 042326 (2018)

AH and Wolfgang Lechner
PRA **100** 032110 (2019)

Quantum phase transitions in p-spin model

Yuki Susa et al *PRA* **98** 042326 (2018)

First-order quantum **phase transition** in p-spin model

Exponential closing of minimal gap

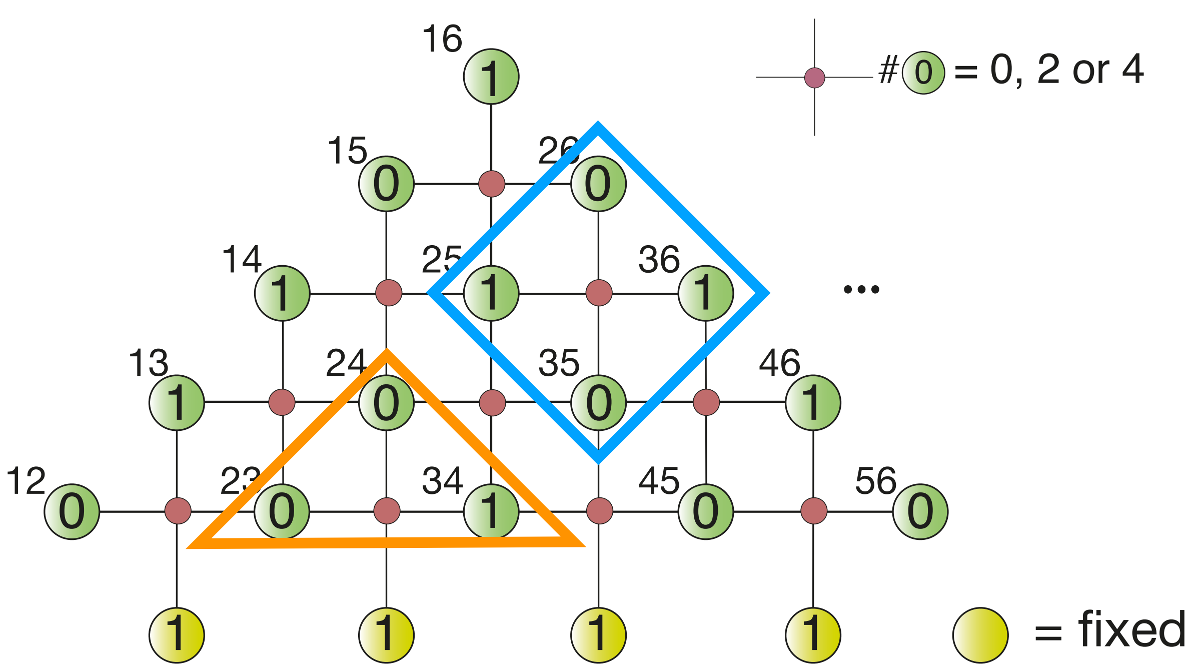
$$H_P = -N\left(\frac{1}{N}\sum_{i=1}^{N}\sigma_i^z\right)^p$$

LHZ has **disordered** and **ordered** part

$$H_{LHZ,P} = \sum_{k=1}^{N_p} J_k \sigma_k^z - \sum_{l=1}^{N_c} C_l \sigma_{l,n}^z \sigma_{l,e}^z \sigma_{l,s}^z \sigma_{l,w}^z$$

p=4 $E_4(m) = -C\left(N_p - \sqrt{1+8N_p} + 2\right) m^4$

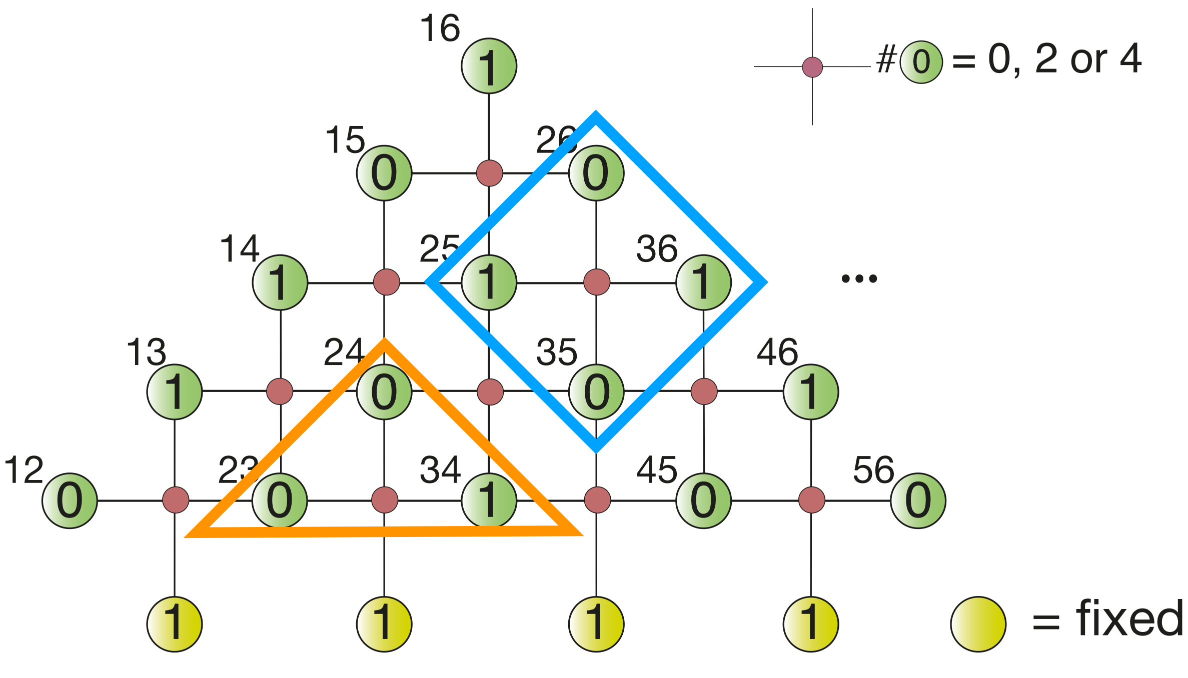
p=3 $E_3(m) = -C\left(\sqrt{0.25+2N_p} - 1.5\right) m^3$



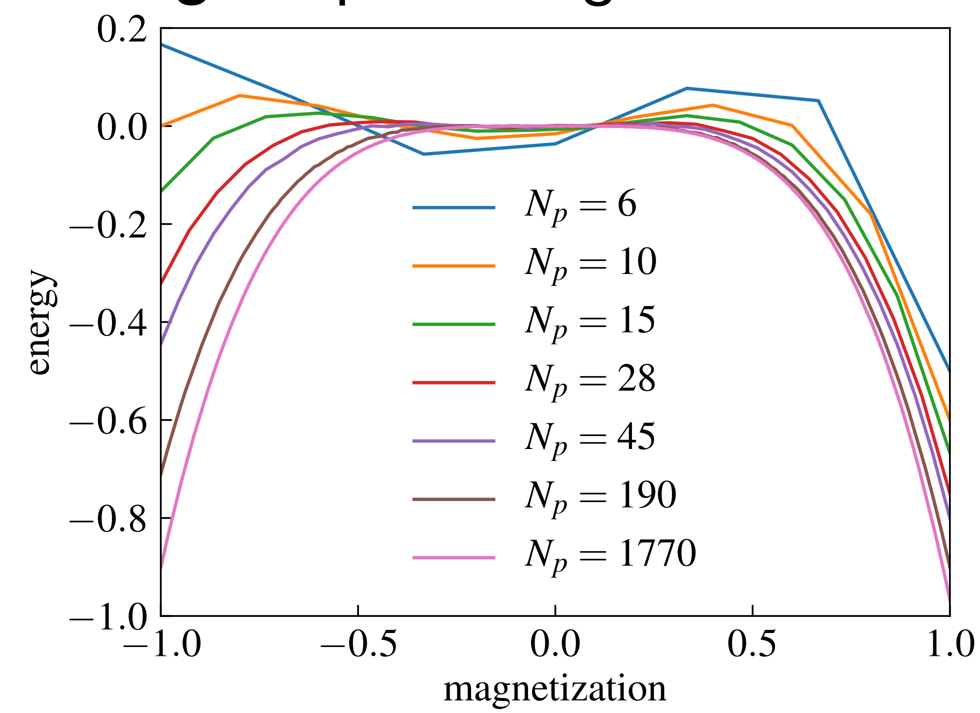
Calculation of constraint energies:

p=4: $E_4(m) = -C\left(N_p - \sqrt{1+8N_p} + 2\right) m^4$

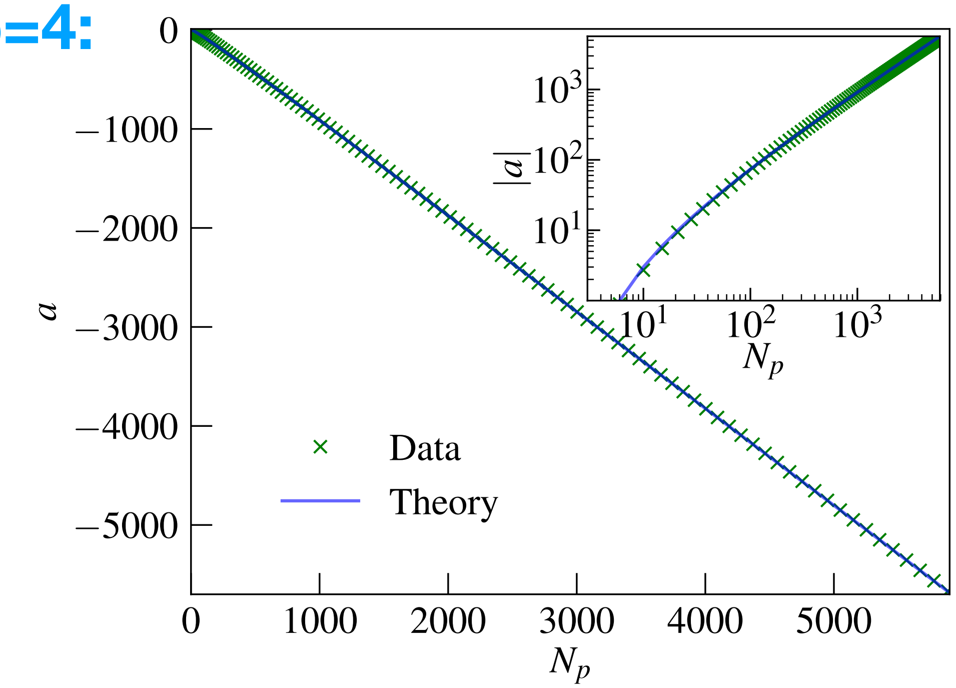
p=3: $E_3(m) = -C\left(\sqrt{0.25+2N_p} - 1.5\right) m^3$



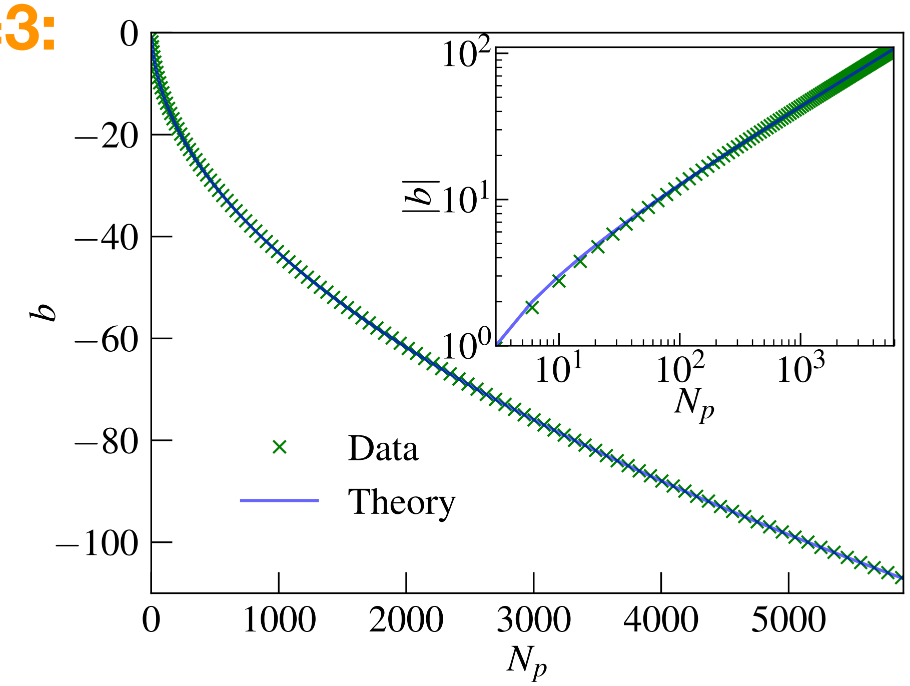
Shuffling of spin configurations in LHZ



p=4:



p=3:



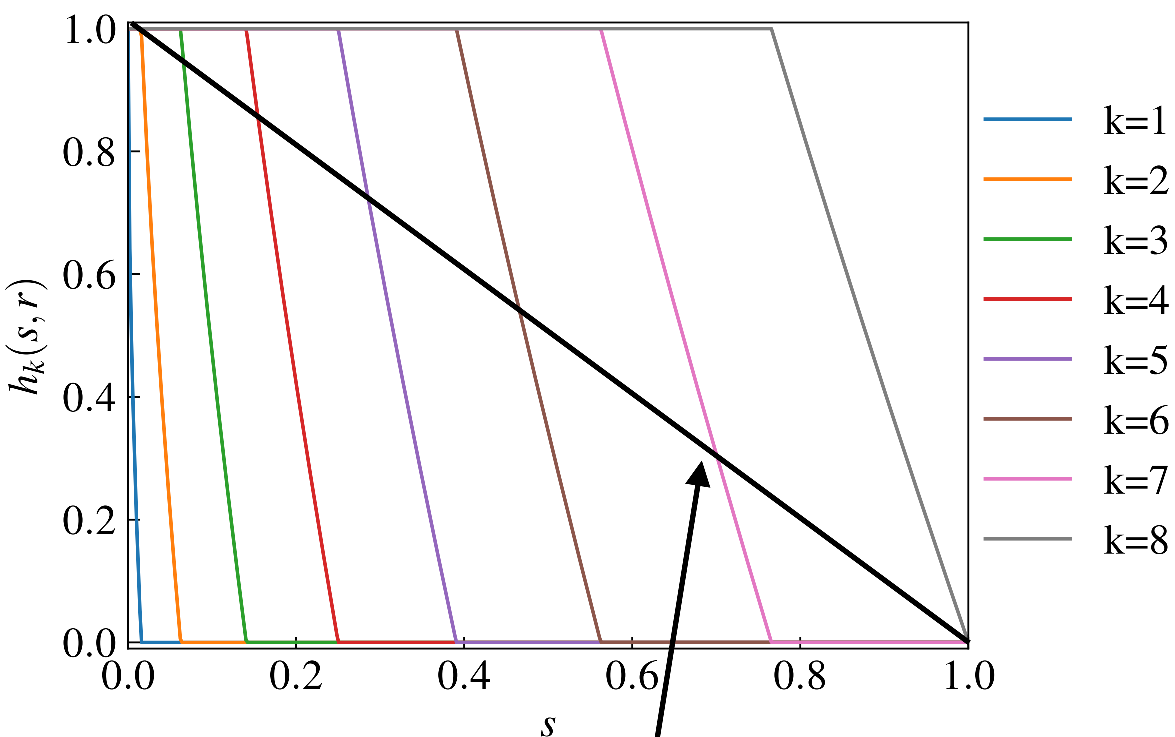
<u>Idea:</u> Inhomogeneous driving of Transverse Field

$$\mathcal{H}_{\mathrm{LHZ}}(s,r) = s\mathcal{H}_P(s) - \sum_{k=1}^{N_p} h_k(s,r)\sigma_k^x$$

normalized time: $s = t/t_f$

Additional new parameter: $\tau = s^r$

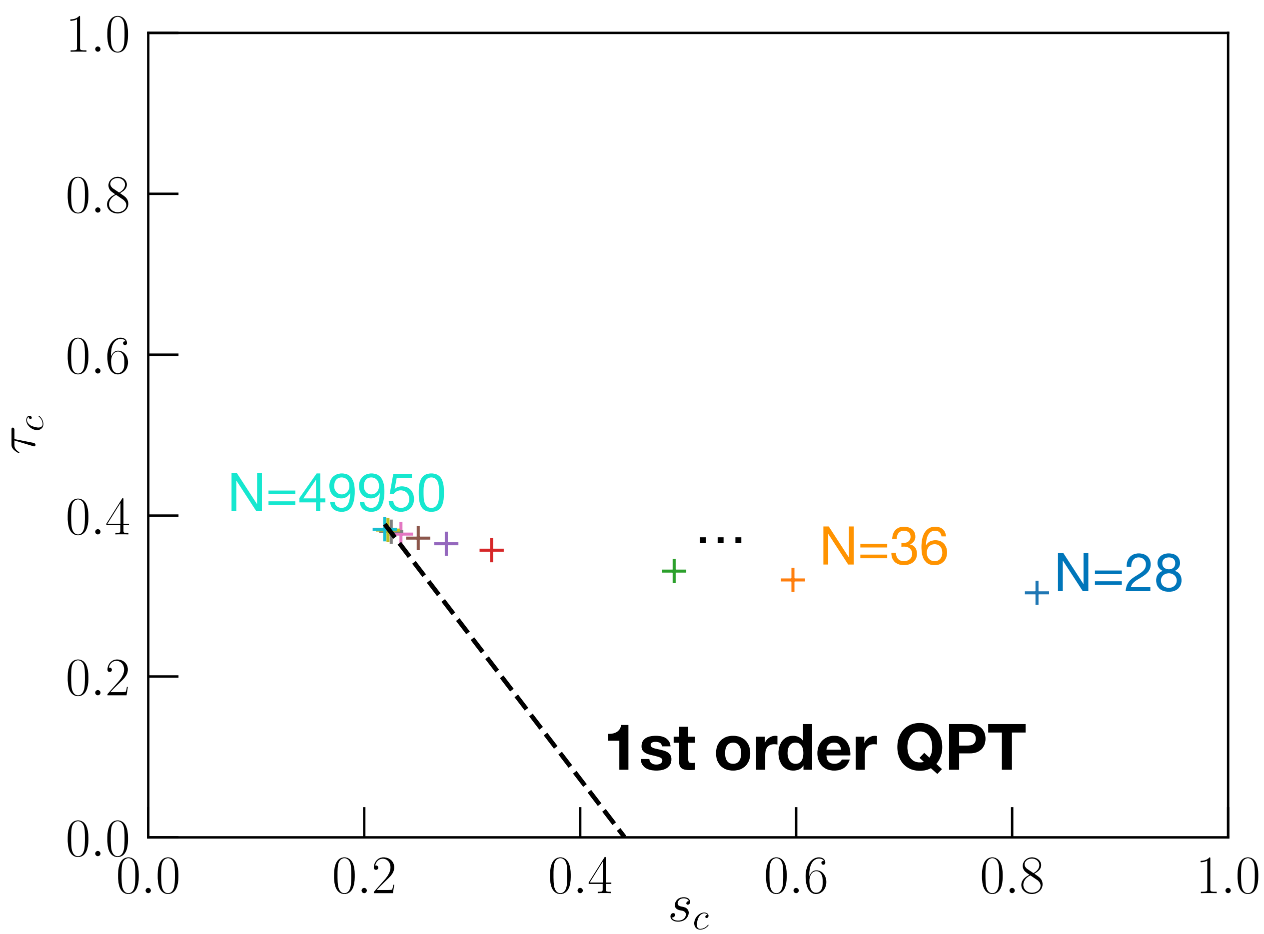
Driver term



standard quantum annealing

homogeneous driving

Two-dimensional phase diagram:



$$\mathcal{H}_{\mathrm{LHZ}}(s,r) = s\mathcal{H}_P(s) - \sum_{k=1}^{N_p} h_k(s,r)\sigma_k^x$$

Free energy of LHZ

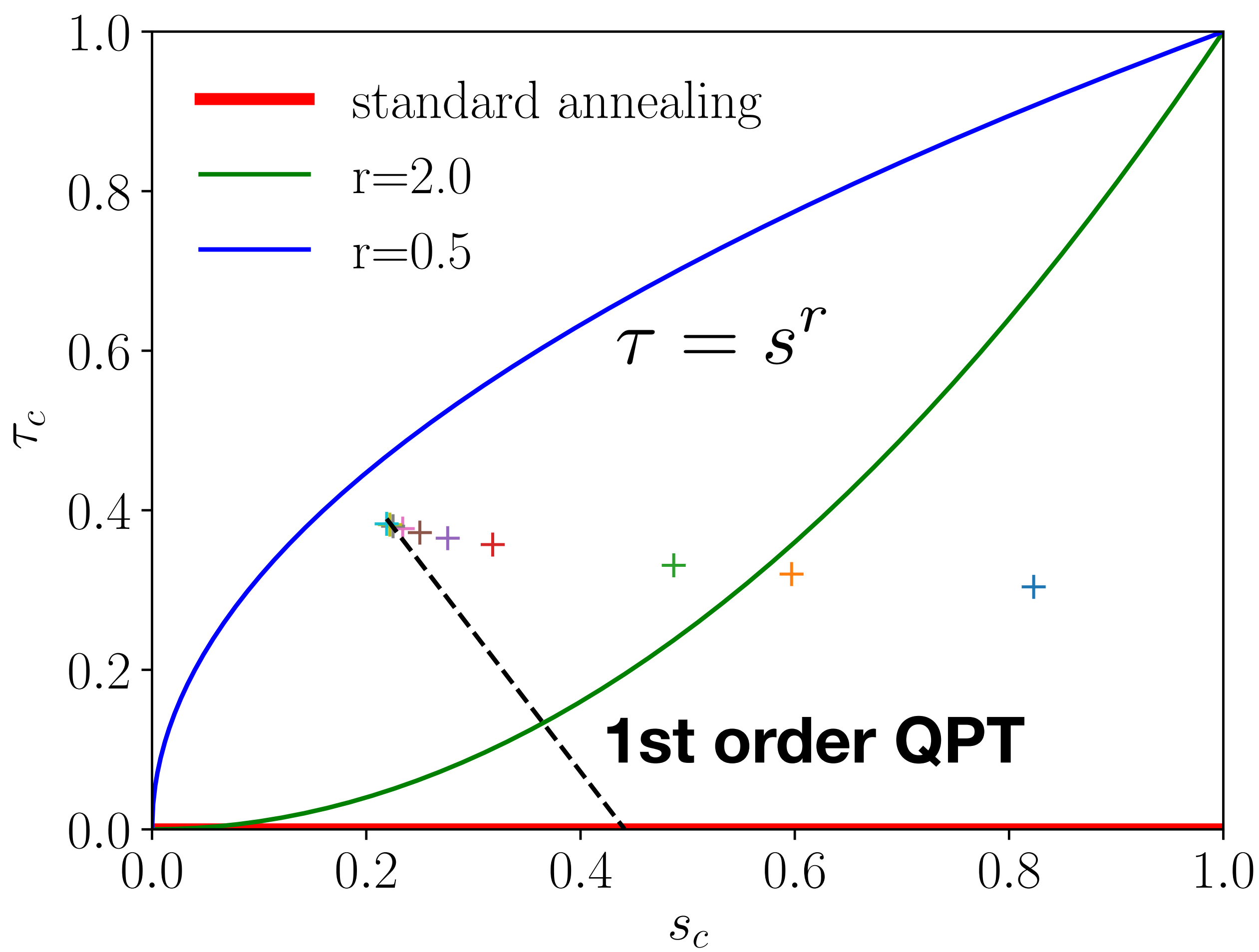
$$f(m,s,\tau,C,J) = 3sCm^4 + \Big[-\tau s(4Cm^3+J) \\ -(1-\tau)\sqrt{s^2(4Cm^3+J)^2+1}\Big]$$

Critical coefficients of LHZ

▷ $m_c \approx 0.679795,\ s_c \approx 0.219232,\ \tau_c \approx 0.38911$

Inhomogeneous driving **extends** phase diagram

Two-dimensional phase diagram:



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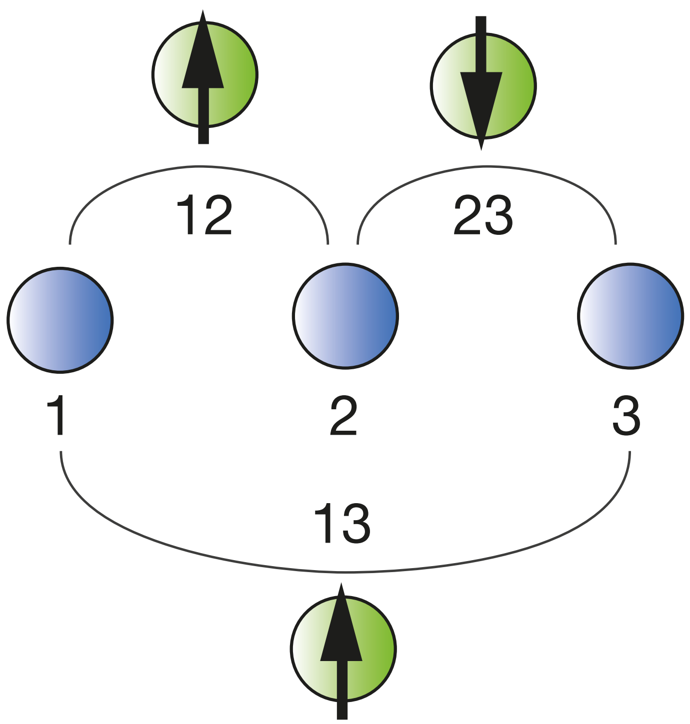
Inhomogeneous driving **extends** phase diagram

Find **path** to avoid 1st-order QPTs

Conditions on closed loops



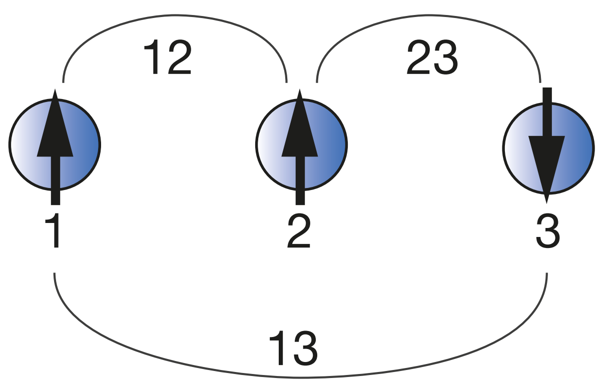
$\sigma_z^{(1)}$	$\sigma_z^{(2)}$	$\sigma_z^{(3)}$	$\hat{\sigma}_z^{(12)}$	$\hat{\sigma}_z^{(13)}$	$\hat{\sigma}_z^{(23)}$
1	1	1	1	1	1
0	0	0	1	1	1
0	0	1	1	0	0
	…			…	
			1	1	0

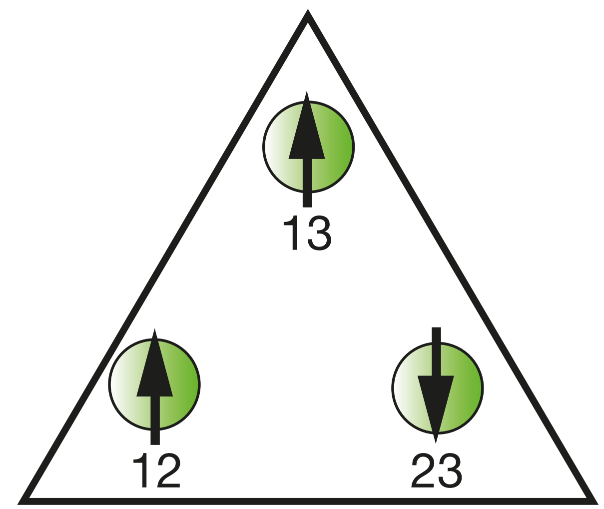
In **each closed loop**, the number of **spin-down** has to be an **even** number or 0.

Lattice gauge encoding

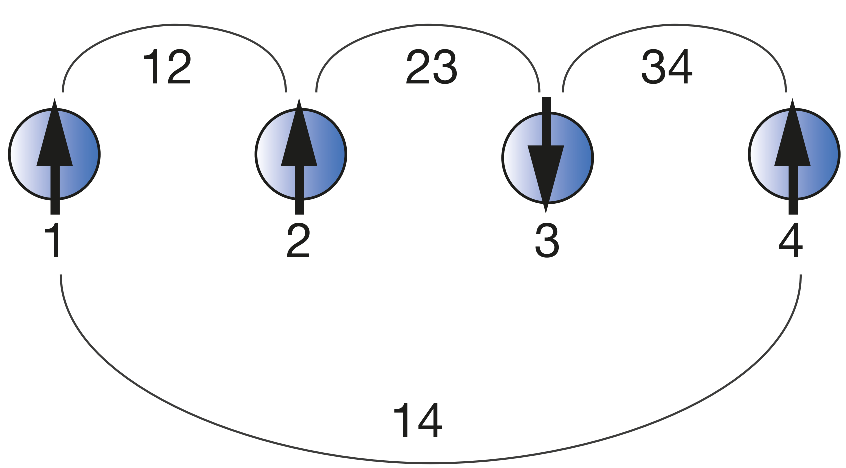
Logical qubits

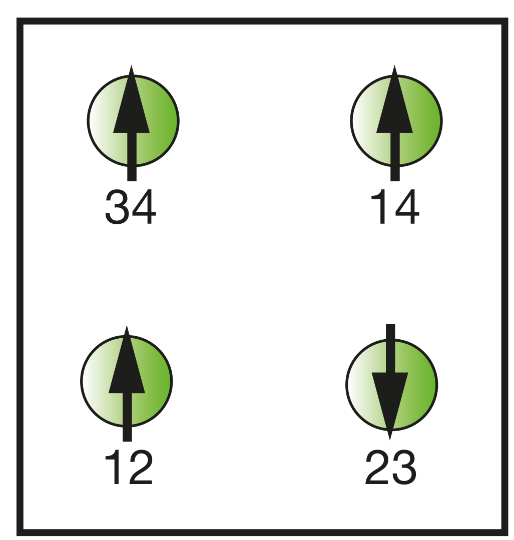
Physical qubits





$$C_l = -C\hat{\sigma}_z^{(12)}\hat{\sigma}_z^{(23)}\hat{\sigma}_z^{(13)}$$

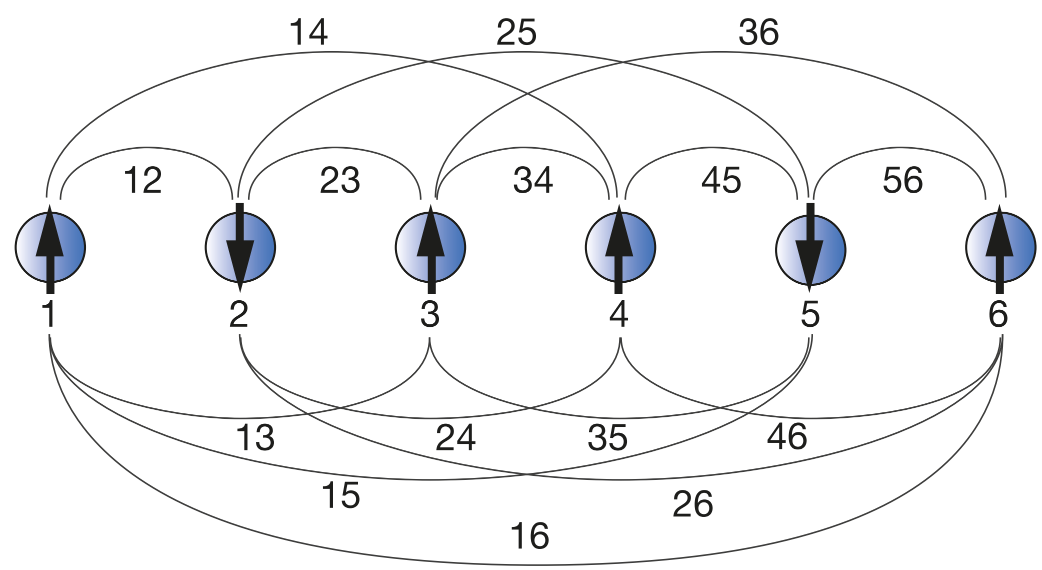




$$C_l = -C\hat{\sigma}_z^{(12)}\hat{\sigma}_z^{(23)}\hat{\sigma}_z^{(34)}\hat{\sigma}_z^{(14)}$$

Lattice gauge encoding

Logical qubits



Physical qubits