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Deep Learning for Photoacoustic Tomography

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Inverse Problem

An inverse problem is a process of calculating causes from observations.

Inverse problems usually occur if I can not observe something directly for example

1. computed tomography
2. deconvolution
3. parameter estimation of differential equations

Inverse Problem

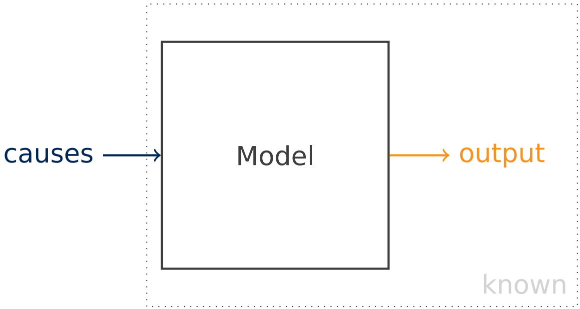


Figure: Idealized inverse problem. We know the model and measure the output. Then we want to calculate the causes.

Toy Example

For two numbers find the quadratic polynomial that has them as roots.

$$-1,\, 2 \Rightarrow (x+1)(x-2) = x^2 - x - 2 \quad \text{or}$$
$$21x^2 - 21x - 42$$

If I do not have additional informations (e.g. coefficient of x^2 is 1), then the solution is **not unique**.

Ill Posed

Definition

A problem is ill posed if one of the following conditions is **violated**

1. there is a solution
2. the solution is unique
3. the solution depends continuously on the observations

Inverse problems are ill posed.

Photoacoustic Tomography

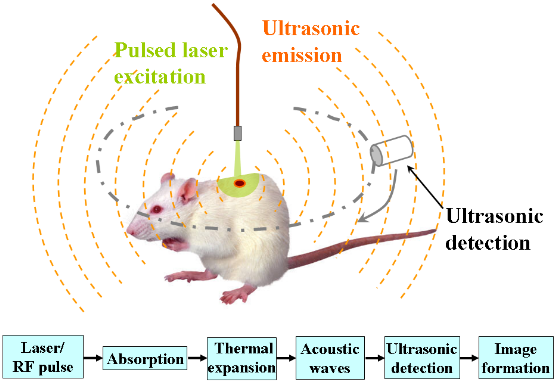
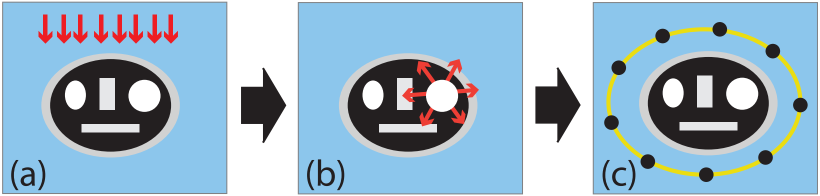


Figure: Principle of photoacoustic tomography.

Photoacoustic Tomography

Let $p_0 \colon \mathbb{R}^2 \to \mathbb{R}$ denote the PA source (initial pressure distribution). The induced pressure wave satisfies the following equation

$$\partial_t^2 p(\mathbf{r}, t) - \Delta_{\mathbf{r}} p(\mathbf{r}, t) = 0$$
$$p(\mathbf{r}, 0) = p_0(\mathbf{r})$$



Photoacoustic Tomography

number of sensors is limited $\Rightarrow$ ill posed

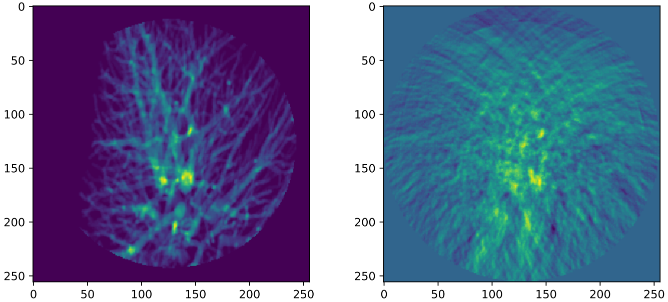
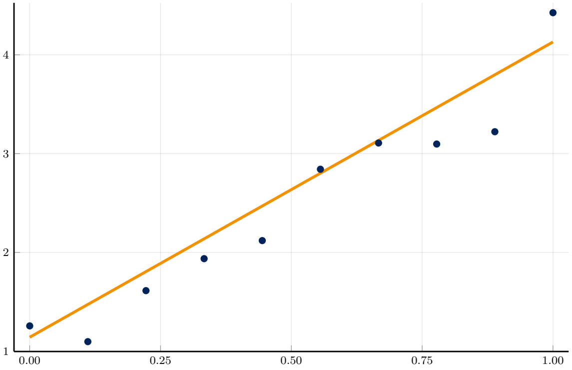


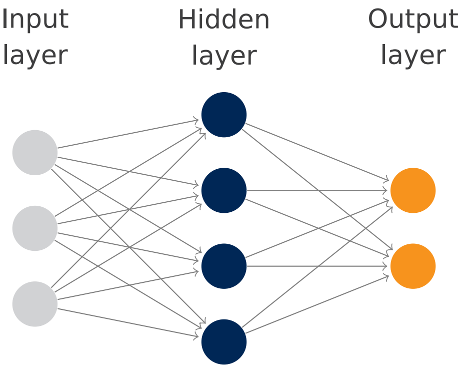
Figure: Left. True image. Right. "Naive" reconstruction.

Neural Networks

- finding parameterized function (learning)
- minimize error on data
- Simple case: **Linear Regression**



Neural Networks



Neural Networks

$$\mathbf{o}^{(1)} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\mathbf{o}^{(2)} = \sigma_1 \left(\begin{bmatrix} W^{(1)}_{1,1} & W^{(1)}_{1,2} & W^{(1)}_{1,3} \\ W^{(1)}_{2,1} & W^{(1)}_{2,2} & W^{(1)}_{2,3} \\ W^{(1)}_{3,1} & W^{(1)}_{3,2} & W^{(1)}_{3,3} \\ W^{(1)}_{4,1} & W^{(1)}_{4,2} & W^{(1)}_{4,3} \end{bmatrix} \mathbf{o}^{(1)} + \begin{bmatrix} b^{(1)}_1 \\ b^{(1)}_2 \\ b^{(1)}_3 \\ b^{(1)}_4 \end{bmatrix} \right)$$

$$\mathbf{o}^{(3)} = \sigma_2 \left(\begin{bmatrix} W^{(2)}_{1,1} & W^{(2)}_{1,2} & W^{(2)}_{1,3} & W^{(2)}_{1,4} \\ W^{(2)}_{2,1} & W^{(2)}_{2,2} & W^{(2)}_{2,3} & W^{(2)}_{2,4} \end{bmatrix} \mathbf{o}^{(2)} + \begin{bmatrix} b^{(2)}_1 \\ b^{(2)}_2 \end{bmatrix} \right)$$

Deep Learning

- arbitrary weights too complex for images
- use **convolutions**
- non-linear layers (Max Pooling,...)
- "many" layers

Deep Learning

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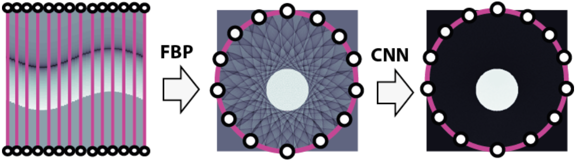
apply network to simple/naive reconstruction $\rightarrow$ post-processing approach



Deep Learning

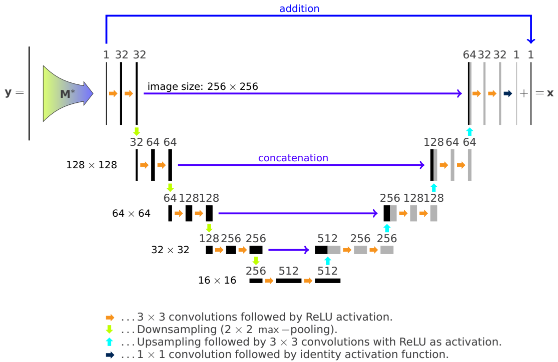


Figure: U-net architecture used in Antholzer, Schwab, Haltmeier 2019. Deep learning from photoacoustic tomography from sparse data *Inverse problems in science and engineering*.

Deep Learning

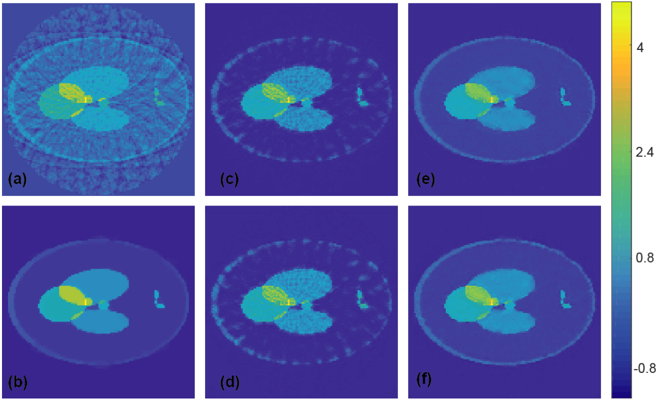


Figure: Left. Reference methods. Middle. CNN with wrong training data. Right. CNN with right training data.

Trained Regularizer

We consider the following problem

$$\text{Estimate } x \in X \text{ from data } y_\delta = \mathcal{A}x + \xi_\delta \tag{1}$$

Do this by solving the following problem

$$\mathcal{T}_{\alpha,y_\delta}(x) := \|\mathcal{A}x - y_\delta\|_2^2 + \alpha\mathcal{R}(x) \to \min_{x\in X} \tag{2}$$

Tikhonov regularization but now $\mathcal{R}$ is a neural network.

[Li/Schwab/Antholzer/Haltmeier 2018]
NETT: Solving Inverse Problems with Deep Neural Networks.
arXiv:1802.00092 (in revision)

Trained Regularizer

[Antholzer/Schwab/Bauer-Marschallinger/Burgholzer/Haltmeier 2019]
NETT regularization for compressed sensing photoacoustic tomography. SPIE proceeding.

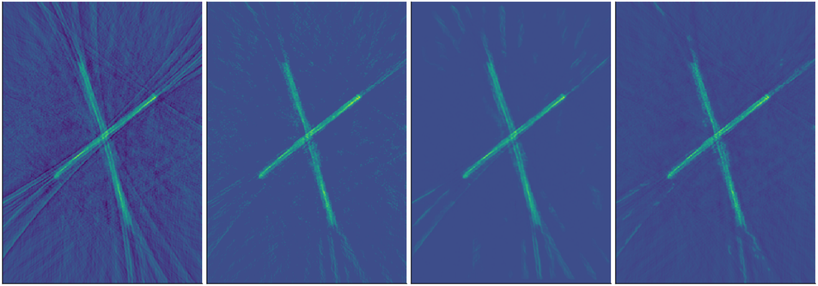


Figure: From left to right: FBP, ℓ_1-minimization, U-net and NETT.

Summary and Conclusion

1. inverse problem (**ill posed**)
2. photoacoustic tomography (**medical imaging, undersampled**)
3. neural networks (**regression, layers, convolutions**)
4. neural networks as post-processing
5. neural networks as regularizers

- neural network useful tool for inverse problems
- lacking theory
- NETT $\rightarrow$ theory

Outlook

- Analysis of compressed sensing photoacoustic tomography
  - number of samples
  - uniqueness
  - convergence
  - sparsifying transform
- More theoretical results (convergence, ...)
- Clinical Applications