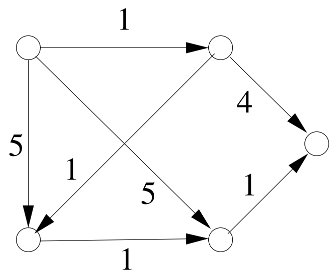
Starred exercises are optional.

1) Let G be the directed graph given by the matrix $A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$

- Give a representation of G in terms of sets of vertices and edges, and draw G.
- Determine for each $0 \leq n \leq 6$, the number of paths of length exactly n from a node in G to itself.
  (For $n = 6$ this is already non-trivial.)

2) Use Floyd's algorithm to compute the shortest paths between any two nodes in the weighted digraph G given by



in two ways: first by taking the top–left node for the first row/column of the matrix B, enumerating nodes clockwise, and next for the matrix B' obtained by doing the same but enumerating nodes counterclockwise. Give apart from the first matrices B and B' also the third matrices C and C' and final matrices D and D'.
(Suggest to use the WebApp demonstrated in the lecture for this.)

Note that B and B' have the same elements but permuted. Does the same hold for C and C', respectively D and D'? Why (not)?

3)
- Suppose G is a directed graph having v among its vertices. Argue that any path p in G has a unique *v-split*, i.e. can be written uniquely as a concatenation of paths $p_1, \ldots, p_n$ for some n, such that p_i starts (ends) at v and is non-empty if $i > 1$ ($i < n$), and v only occurs as start/end of the p_i (not as an intermediate node). For instance, the v-split of the path $(u, v, w, v, w, w, v, w, x)$ is $(u, v), (v, w, v), (v, w, w, v), (v, w, x)$.
- Suppose G also has another vertex w. Does w-splitting (each path in) the v-split of a path p yield the same result (as list of paths) as v-splitting the w-split of p? Compute both for the path given in the previous item.

4∗) Adapt Floyd's algorithm to return the *number* of non-empty paths between any pair of nodes. In case there are infinitely many paths between a given pair of nodes, the result should be ∞. For instance, in the graph of exercise 2 above the number of non-empty paths from the top–left node to the rightmost node is 4. For another example, for the graph having two nodes v, w and edges (v, w) and (w, v), the initial and final matrices should be $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} \infty & \infty \\ \infty & \infty \end{pmatrix}$