Starred exercises are optional.

1) a) Determine whether or not the following language is recursive:

$$L = \{M\#xy \mid x, y \in \{0,1\}^* \text{ with } \ell(x) = \ell(y) \text{ and } x \text{ is accepted by TM } M\}$$

(Recall ℓ denotes the length function.) To show recursiveness describe a total TM (or terminating program) accepting the language. To show non-recursiveness either use diagonalisation directly, or give a reduction from a language known to be non-recursive (HP,MP).

b) Give an example showing that the 'reducible' relation $\leq$ is not anti-symmetric. Describe in words the equivalence classes of the equivalence relation induced by $\leq$.

2)
- Let G be a directed graph with nodes $\{a, b, c, d\}$ and labeled edges

$$\{(a, 4, b), (a, 3, c), (a, 1, d), (b, 1, a), (b, 3, d), (c, 1, b), (d, 1, c)\}$$

where each triple describes the start of the edge, the weight, and the end of the edge. Use Floyd's algorithm to compute the distances between all nodes. Give the start matrix and all intermediate matrices.

- Let G be the undirected graph with nodes $\{a, b, c, d, e, f, g\}$ and edges

$$\{a, b\}, \{a, e\}, \{b, e\}, \{b, f\}, \{c, d\}, \{c, g\}, \{d, g\}, \{e, f\}$$

The weights of the edges in this order are $1, 2, 3, 1, 2, 1, 3, 2$. Use Kruskal's algorithm to compute a spanning forest with minimal weight.

3) a) Describe the Chinese Remainder Theorem (Bézout version) in your own words.

b) Consider the two congruence systems

$$\begin{aligned} x &\equiv 5 \pmod 7 \\ x &\equiv 2 \pmod 8 \end{aligned} \qquad \text{and} \qquad \begin{aligned} y &\equiv 3 \pmod 7 \\ y &\equiv 4 \pmod 8 \end{aligned}$$

Show that each system can be solved using the Chinese Remainder Theorem and indicate the corresponding steps taken resulting in the respective solutions for x and y. Furthermore, verify your results.

c) Argue that the following congruence system cannot be solved.

$$\begin{aligned} x &\equiv 2 \pmod 4 \\ x &\equiv 3 \pmod 6 \end{aligned}$$

Does this contradict the Chinese Remainder Theorem?

4*) Same question as exercise 1a) but both for the language

$$L = \{M\#x \mid x \in \{0,1\}^* \text{ where TM } M \text{ halts on } xx\}$$

and for its complement $\sim L$.