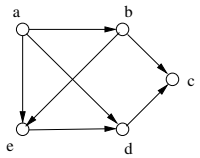
1) Consider the relation R on $\{a, b, c, d, e\}$ whose graph is



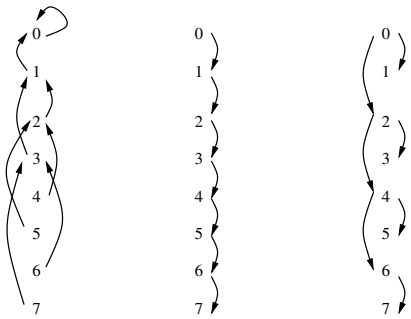
(the graph of 2nd exercise of the 1st Exercise sheet but without weights). Use Warshall's algorithm to compute the transitive closure R^+ of R. Read off from the matrix whether $a\ R^+\ c$ holds, $c\ R^+\ a$ holds, and whether $d\ R^+\ b$ holds. (You may use the WebApp).

Is there a connection between (the matrix of) R^+ and the distances (matrix) computed in the 2nd exercise of the 1st Exercise sheet? (The latter matrix is in the solutions on the site.)

2) Let T be the set of all strings of length at most 4 over letters a,b. For instance, a, ba and $aaab$ are in T, but c, abc and $aabaa$ are not in T. Define the (binary) relation $\sqsubset$ on T by $s \sqsubset t$ if t is obtained by *right-rotating* s, i.e. moving the last letter of s to the front. For instance, $aba \sqsubset aab$ and $baab \sqsubset bbaa$, but neither $aa \sqsubset aaa$ nor $aaab \sqsubset aaba$.

Explicitly write T as set of strings and $\sqsubset$ as a set of pairs in T. Determine both for $\sqsubset$ and its transitive closure $\sqsubset^+$ whether it is reflexive, irreflexive, symmetric, anti-symmetric, transitive.

3) a) Consider the three (initial parts of) graphs having the natural numbers as objects below. For each, write the graph as a relation (set of pairs), and determine whether or not it represents a function on the natural numbers (see slide 22 of the 2nd lecture), assuming the rest of each graph (for numbers > 7) continues in a 'natural' way. If so, give a corresponding Haskell function. If not, explain why not.



b) Vice versa, draw (the initial part) of each of the following relations R_i on the natural numbers as graph and determine whether or not they represent functions.

i. for all n, $n\ R_1\ 2$ if $n \leq 4$, and $n\ R_1\ 3$ if $n \geq 4$;

ii. for all n and m, $n\ R_2\ m$ if $m + 2 = m^2$;

iii. $0\ R_3\ 0$ and for all n and m, $n\ R_3\ (n + m)$ if both $n > 0$ and $(n - 1)\ R_3\ m$.

4∗) Is there a *unique* function f on the natural numbers satisfying the following specification: $f(1) = 1$ and for all n, $f(n) = f(\frac{n}{2})$ if n is even, and $f(n) = f(3{\cdot}n{+}1)$ if n is odd and $n > 1$. For instance, to determine $f(5)$ we may compute $f(5) = f(16) = f(8) = f(4) = f(2) = f(1) = 1$. (Hint: Collatz).